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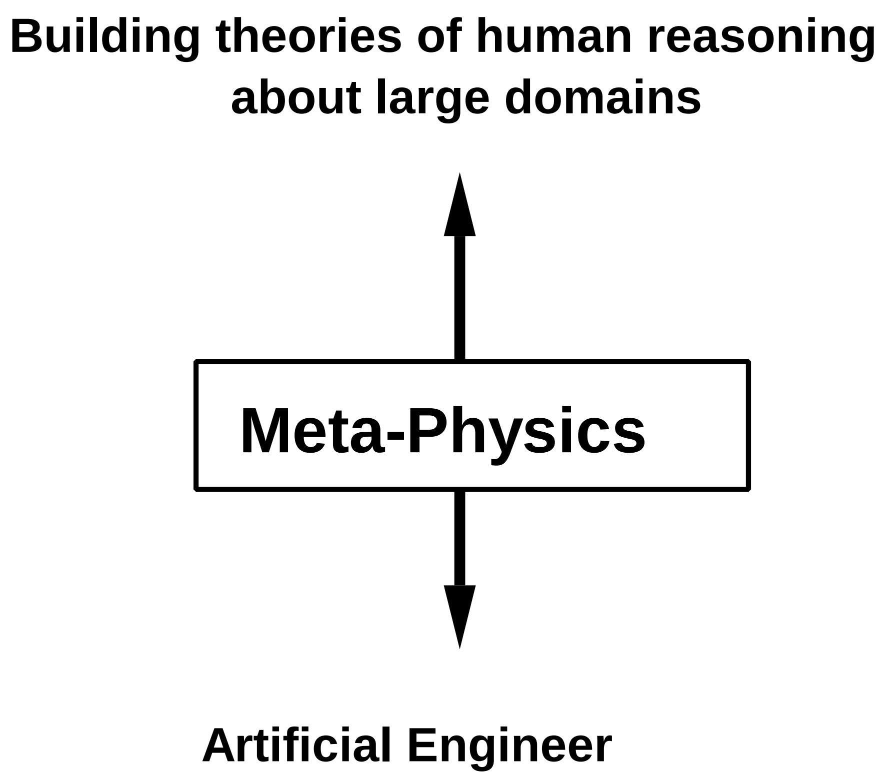
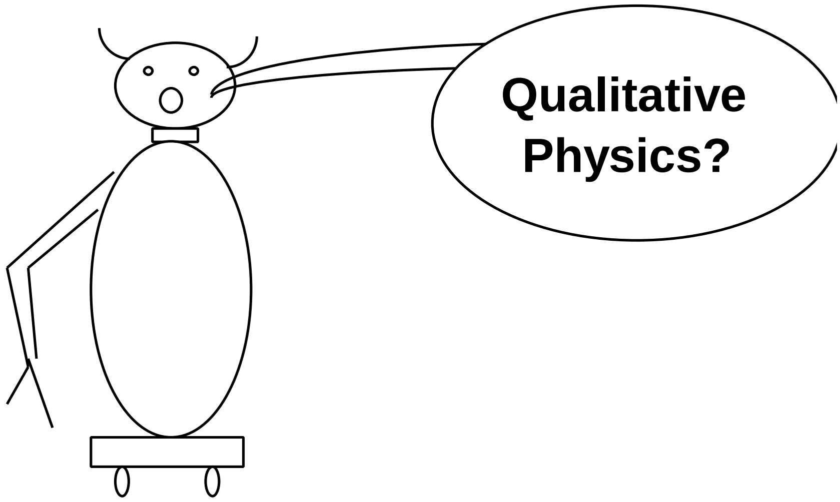
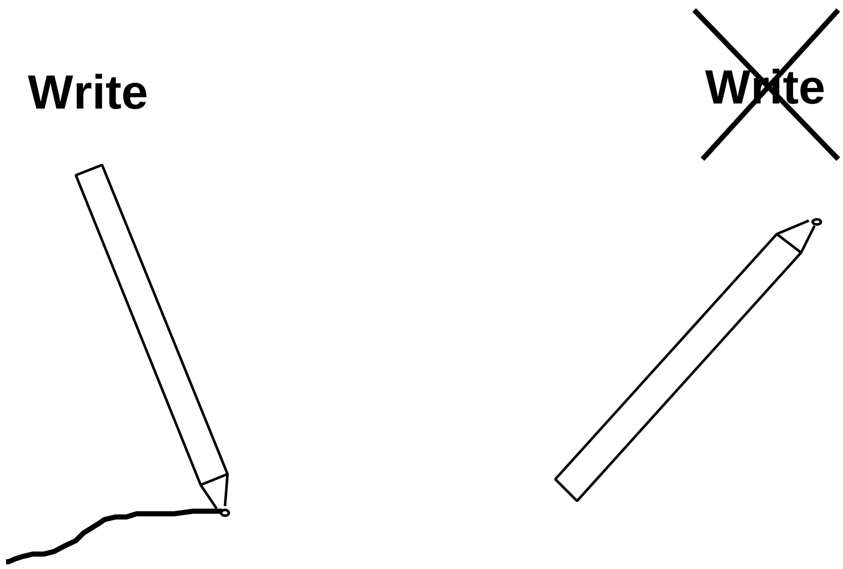
Qualitative Physics

Introduction


Building theories of human reasoning
about large domains
Meta-Physics
Artificial Engineer

World of a Robot

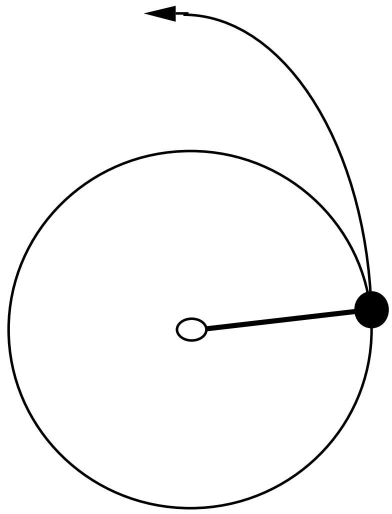




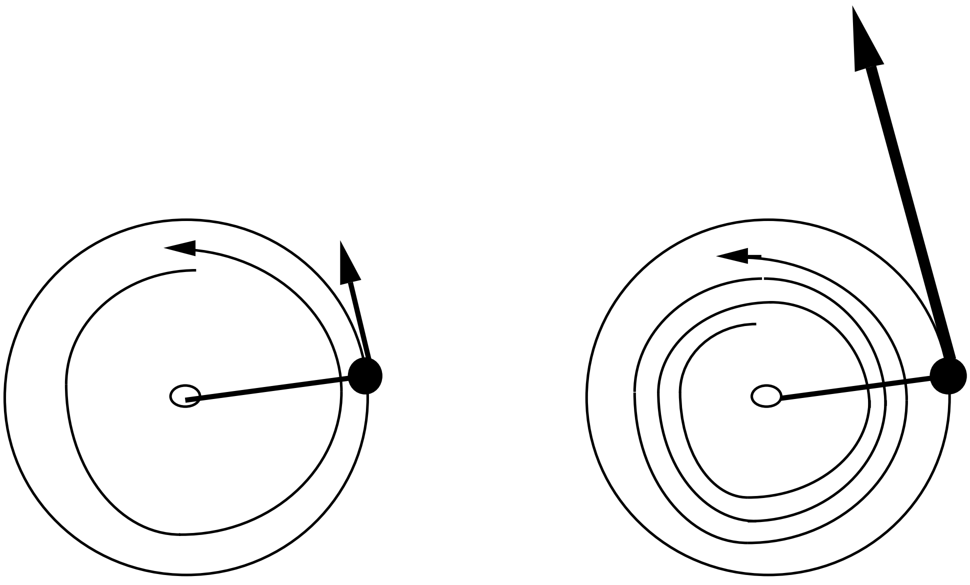
A ball-point pen

Insight: Make tacit knowledge explicit

Simple Problems 1

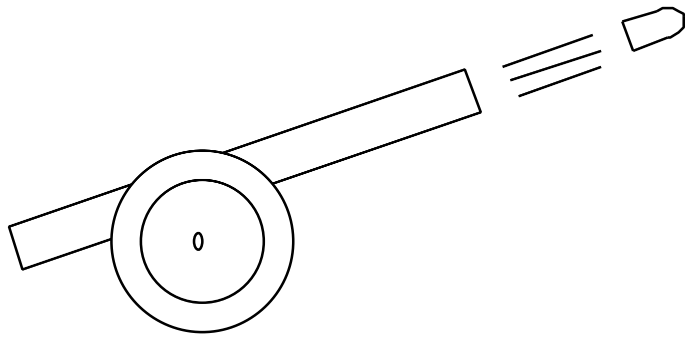


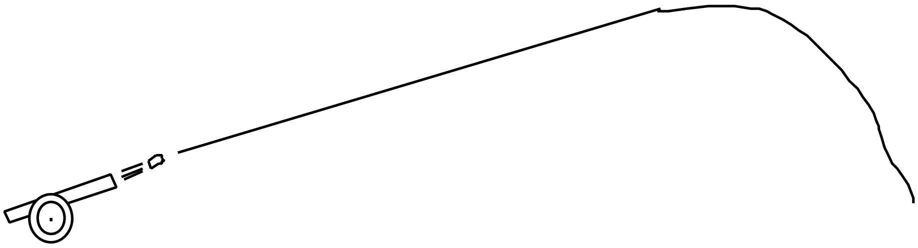
Right trajectory?

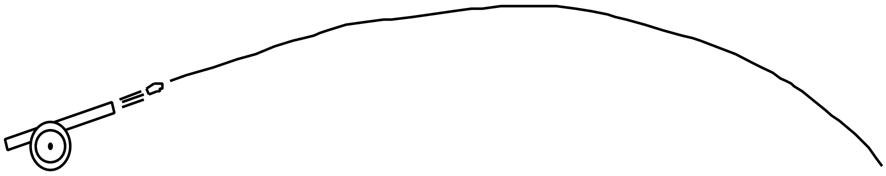


Ten cycles better than one?

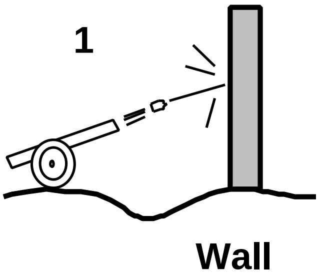
Canons

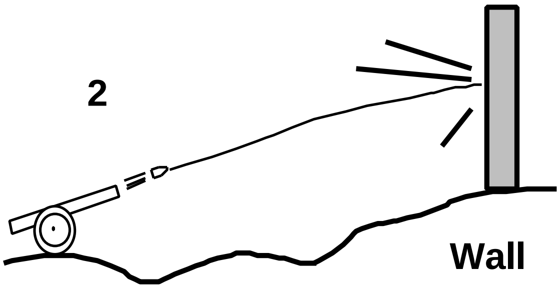






What is the right curve?

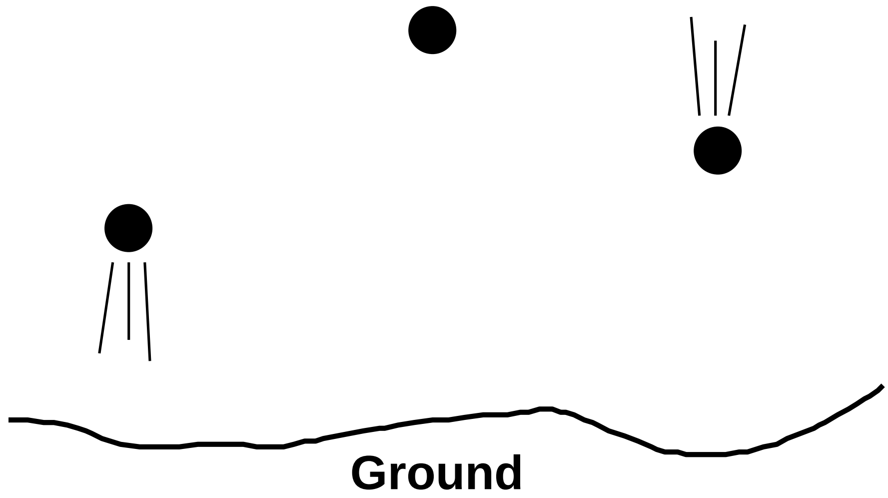




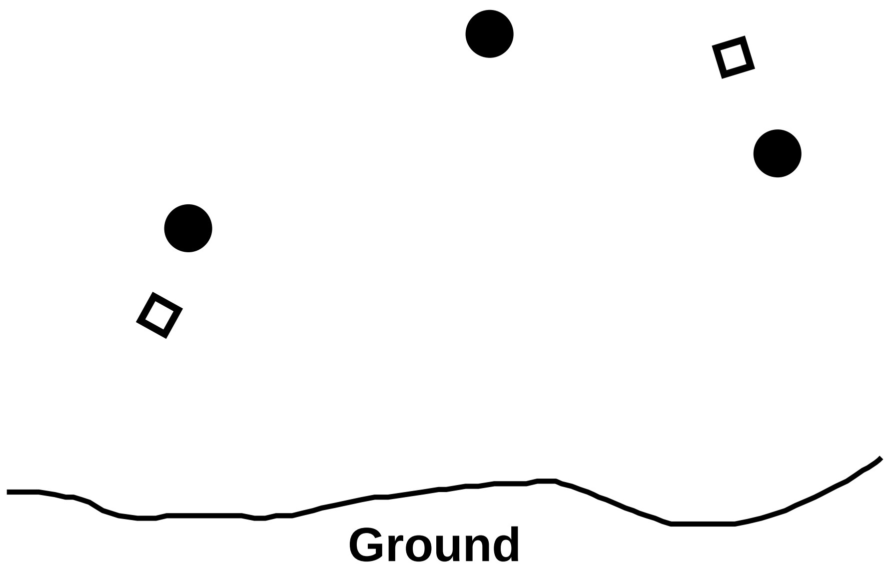
Optimal distance for breaking the wall down?

Pre-History 1

- **What Is Motion?**



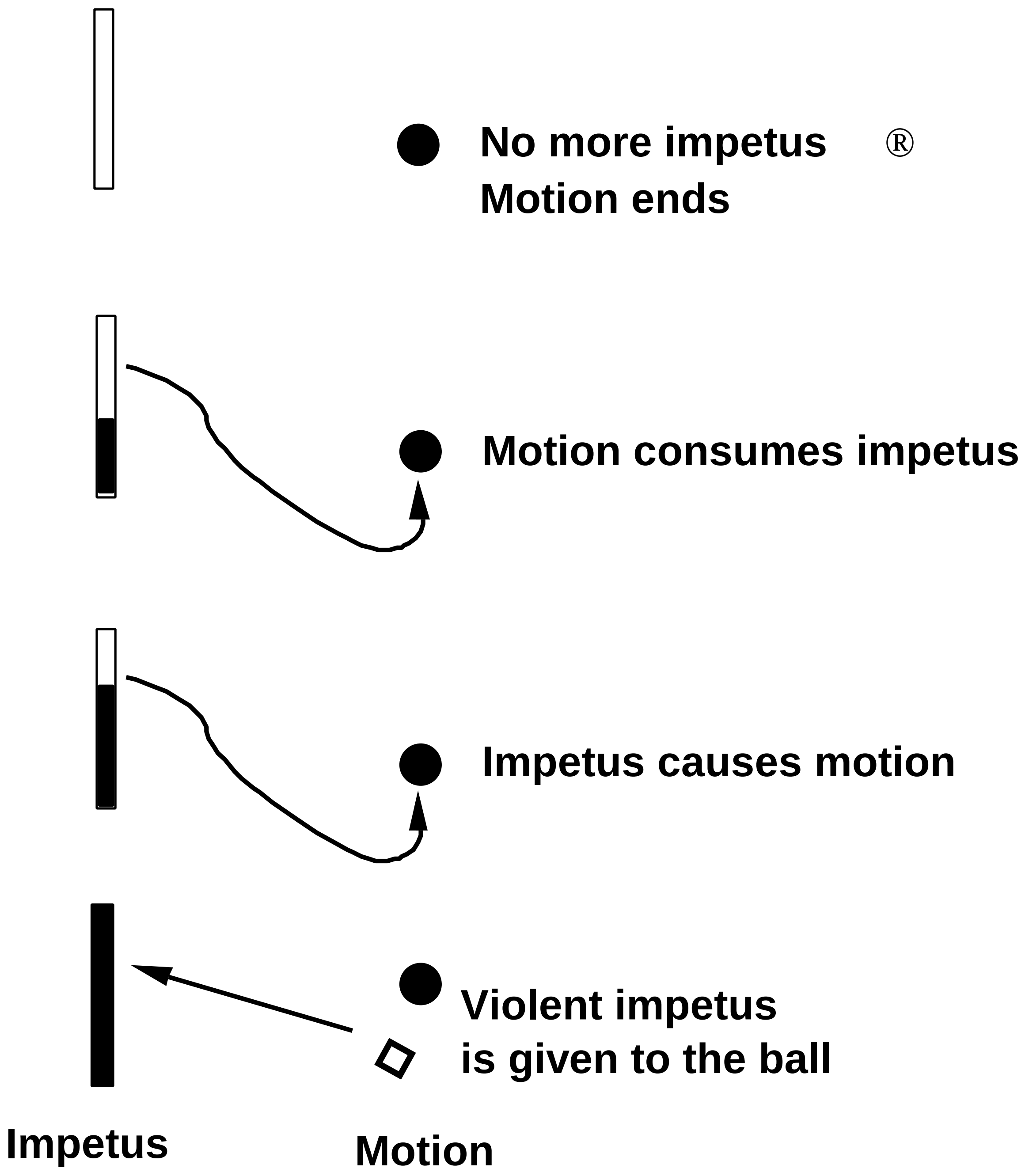
- **What Causes Motion?**



Pre-History 2

• Motion is Caused by Impetus

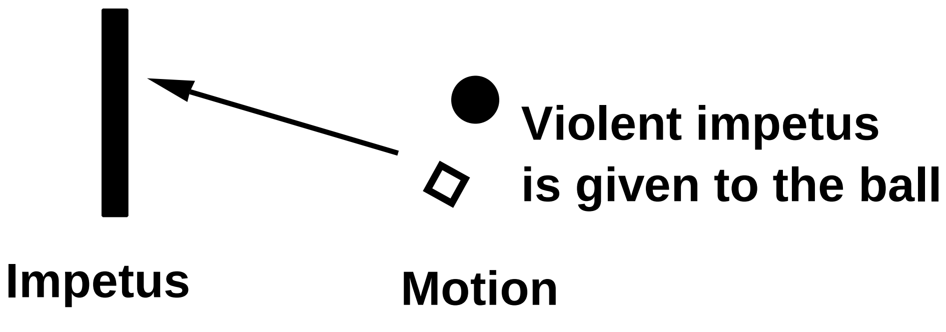
• Motion Consumes Impetus



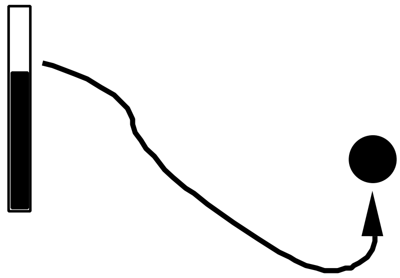
- Insight: No inertia.

(Benedetti, Parisian School)

Pre-History 2

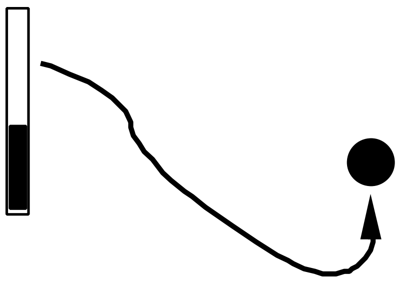


- **Motion is Caused by Impetus**



Impetus causes motion

• **Motion Consumes Impetus**

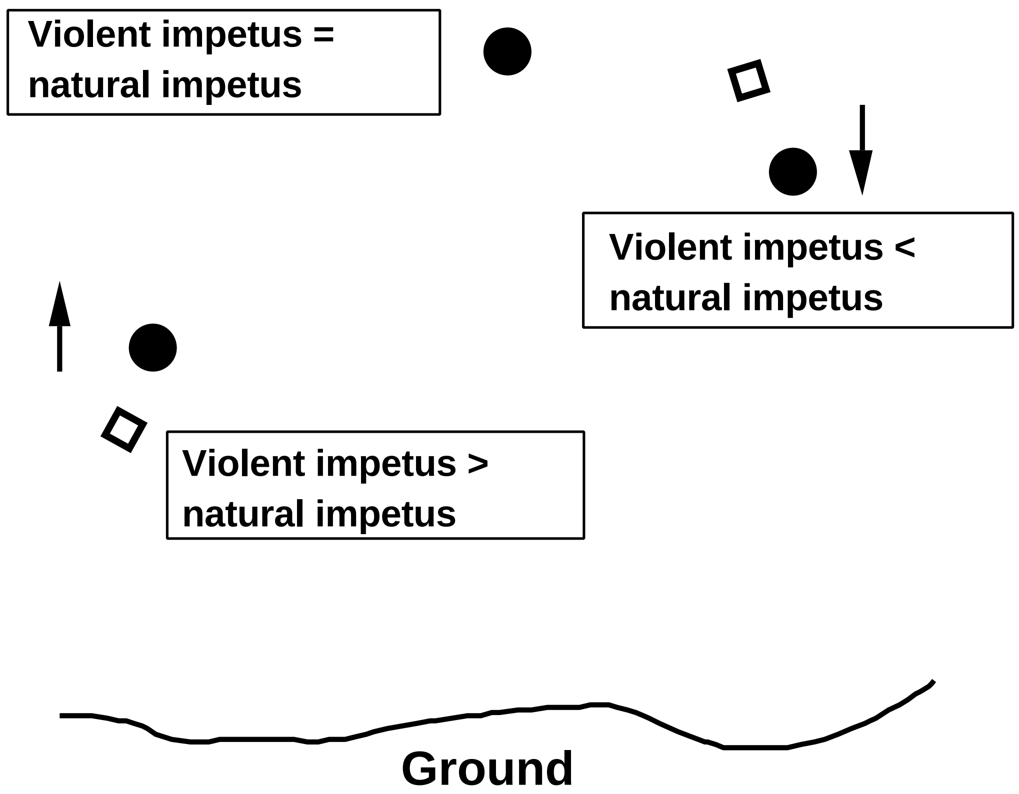


Motion consumes impetus

- Insight: No inertia.

(Benedetti, Parisian School)

Pre-History 3



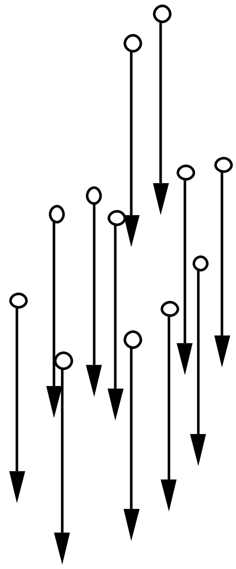
(Young Galileo)

- **Motion still consumes impetus, but gravity permanently gives impetus back.**

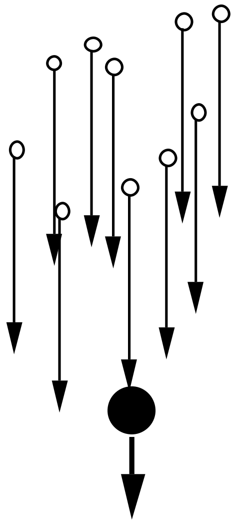
- **Insight: Falling bodies tend to uniform velocity!**

Pre-History 4

- What causes motion? (Descartes)

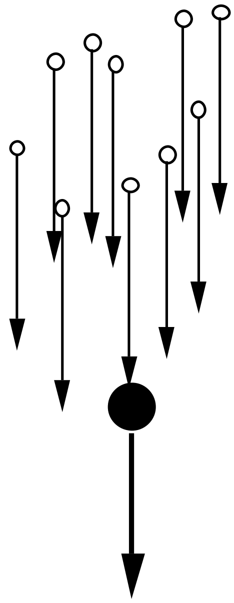


- Unmaterial substance flowing down to Earth.



- Unmaterial substance 'hits' the ball and 'pushes' it down.

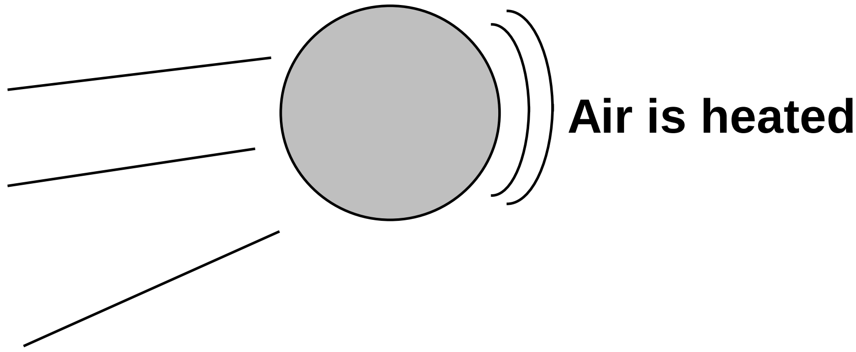
  Velocity increases.



- Velocity increases slower and slower.

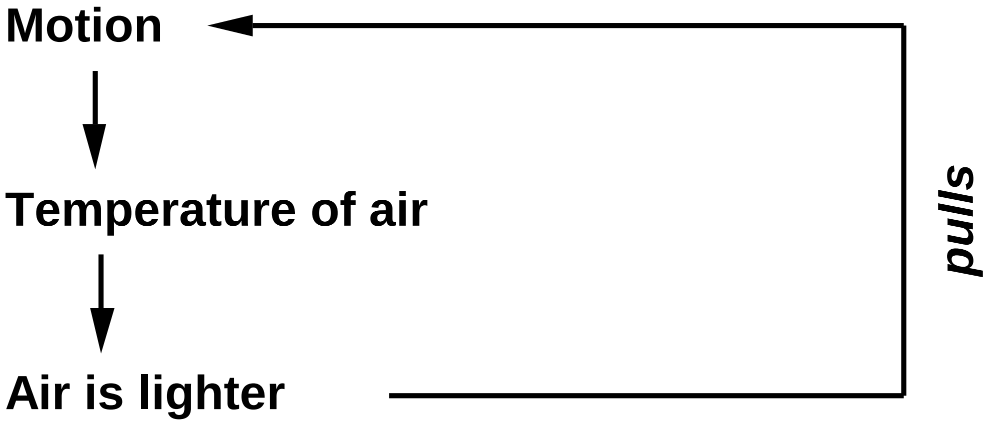
- Falling bodies tend to uniform velocity.

Historical example of 'feedback'

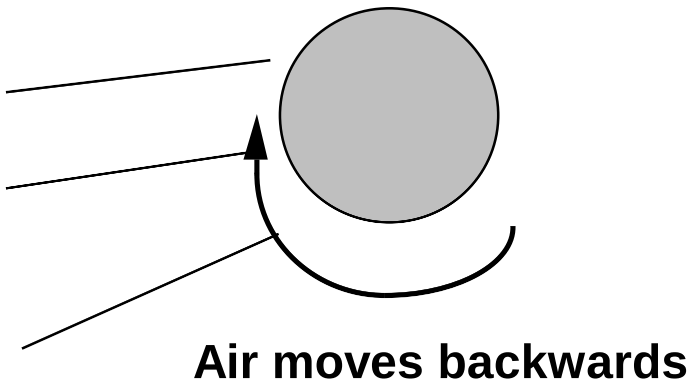


A moving object

"As the object moves, it heats up the region of air which is in front. Thus, air becomes ligther in this region. Thus, the object is pulled and so goes on moving."



Historical example of 'feedback'



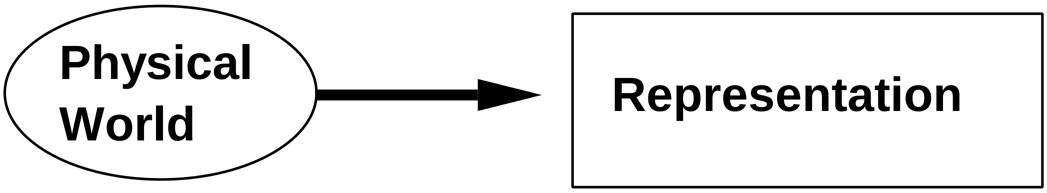
A moving object

"Moreover, part of air in this region may move backwards. Thus, air becomes heavier behind the object, and pushes it."

Insight **: These kinds of feedback are impossible.**

Perpetual motion

Ontology



<u>Commitements:</u>

Individuals and objects.

(e.g. bodies, unmaterial substance, velocity, impetus, …)

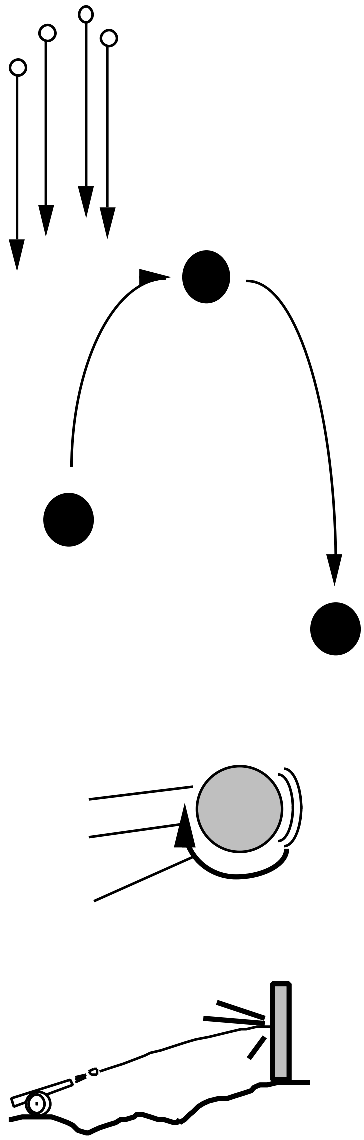
+ Properties of these individuals and objects.

(e.g. pushes, hits, consumes, increases, decreases, uniform…)

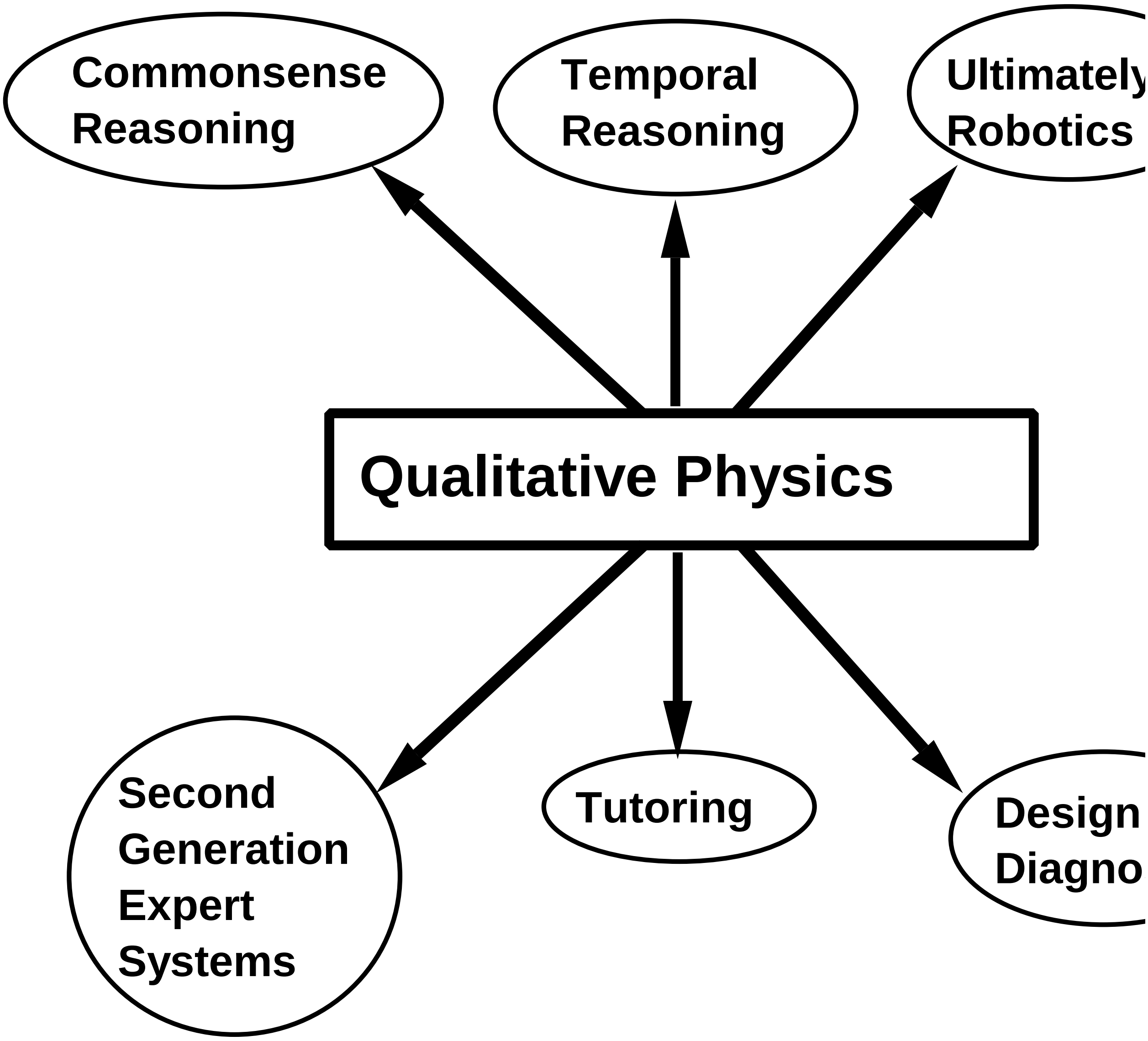
= Ontology of a domain

Primitive concepts

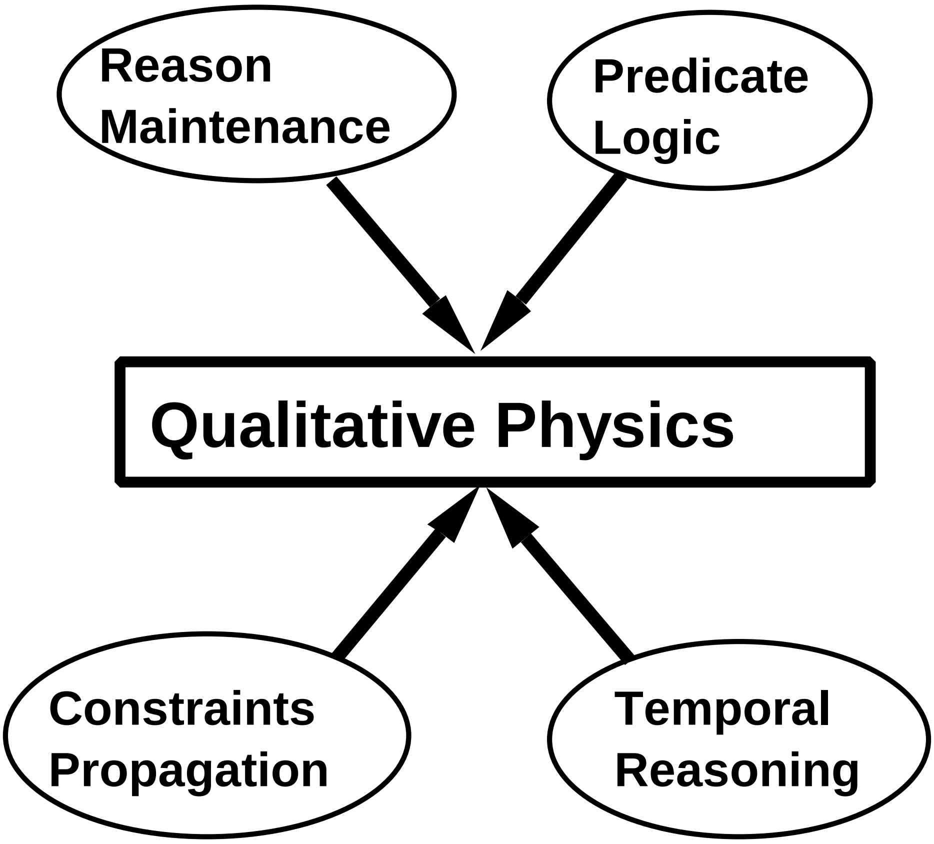
- Ontology
- Changes
- Events
- Time
- Continuity
- Causality
- Teleology
- Quantities



"… Air is lighter …"


Commonsense Reasoning
Temporal Reasoning
Ultimately Robotics
Qualitative Physics
Second Generation Expert Systems
Tutoring
Design
Diagno

Clarification



"Temperature increases"

"Earth is flat"

} ° Fuzzy Logic

Outline
Part 1

- Ontology (for liquids)
  (Hayes)

- Causal reasoning through explicit
  representation of processes
  (Forbus)

- Discovering causality
  (De Kleer, Williams)

- Envisioning: Transition analysis
  (De Kleer, Williams)

An Ontology for Liquids

(Hayes)

What is a liquid?

• No individuation: water in a glass is not an object

• Considering a liquid as the sum of its parts- like a powder -is too sophisticated

® Considering a liquid as contained:

c container ® `inside(c)`

s Interior place ® `capacity(l,s)`

`l` liquid, s interior place
® `amount(l,s)`

Axiom:

`none = amount(l,s) = capacity(s)`

Geometry

- Space contained in another space:

  ® In(s_1,s_2)
- Faces.

- A face of dimension n-1 divides n-dimensional space into exactly two parts:

  `f` face, v half-espace

  ® `toso(f,v)` = the other side of `f`

- `Top` et `Bottom` of a space.

  Axiom:

  `Top(f,v)` É `Bottom(f,toso(f,v))`

- Free space.

- Portal (˜ way out)

  = common face of two spaces

  = surface through which one can pass from (a point in) one space to (a point in) the other.

The fifteen states
of a liquid

- Bulk or divided

- Lazy or energetic
  (cf. natural and violent)

- Supported or unsupported

- If supported, contained or on a surface

- Moving or still (if it is still, it is lazy).

® 15 possibilities

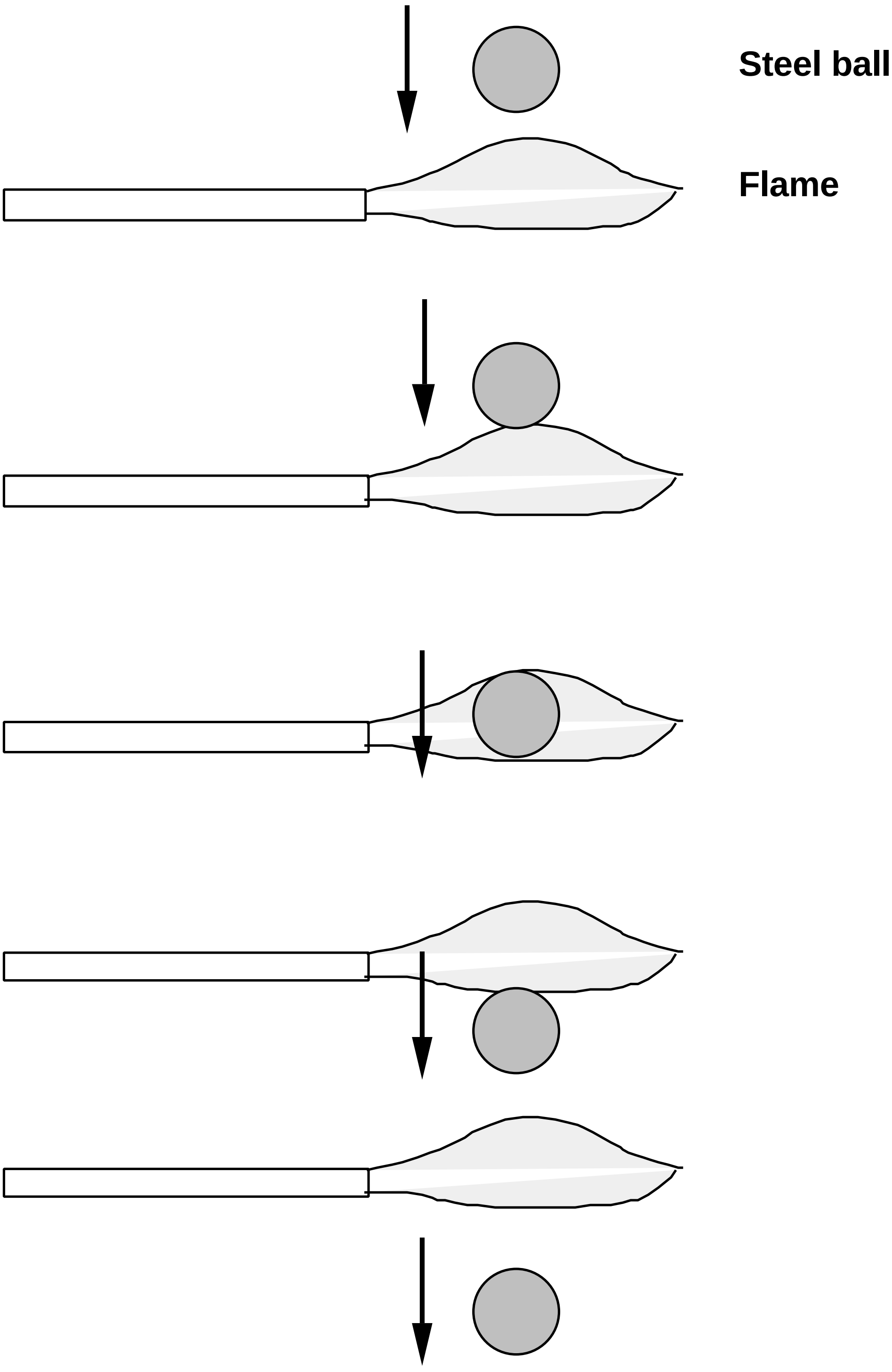
The fifteen states of a liquid

Lazy Still	Lazy Moving	Energetic Moving		
Wet surface	Flowing down a surface (Sloping roof)	Waves lapping shore Jet hitting a surface	2D	Bulk
Contained, in container	Flowing along a channel (River)	Pumped along pipeline	3D	
	Falling column of liquid (Pouring from a jug, waterfall)	Waterspout, fountain, jet from hosepipe	Unsupported	
Dew, drop on a surface			2D	Divided
Mist filling a valley	Mist rolling down a valley	Steam or mist blown along a tube	3D	
Mist, cloud	Rain, shower	Spray, splash, driving rain	Unsupported	

Transitions between states: an example

A steel ball through a flame

(Forbus)



Explicit Causality: Qualitative Process Theory

(Forbus)

The 'History' of the steel ball

The steel ball *moves downwards* , due to the gravity.

It *reaches* the flame.

As it *passes through* the flame, there is a *heat flow* which causes an *increase* of the *temperature* of the steel ball.

Then, the ball *moves away from* the flame.

It goes on *falling down* .

Reasoning about change

What:
- **Position(B)**
- **Temperature(B)**

How:
- **Position(B)** ↘
- **Temperature(B)** ↗

Why:
- **Motion**
- **Heat-flow**

Time

Time ⟷ Change

Events The ball reaches the flame

Episodes As the ball passes through the flame, there is a heat flow which causes an increase of the temperature of the steel ball.

- Events last for an instant
- Episodes have a duration

Events and episodes are intervals. Every interval has a *start* and an *end*. The start and the end of an event are equal, though they are different for an episode.
The start and the end of an episode meet with events.

Insight: There is a partial ordering of starts and ends of intervals.

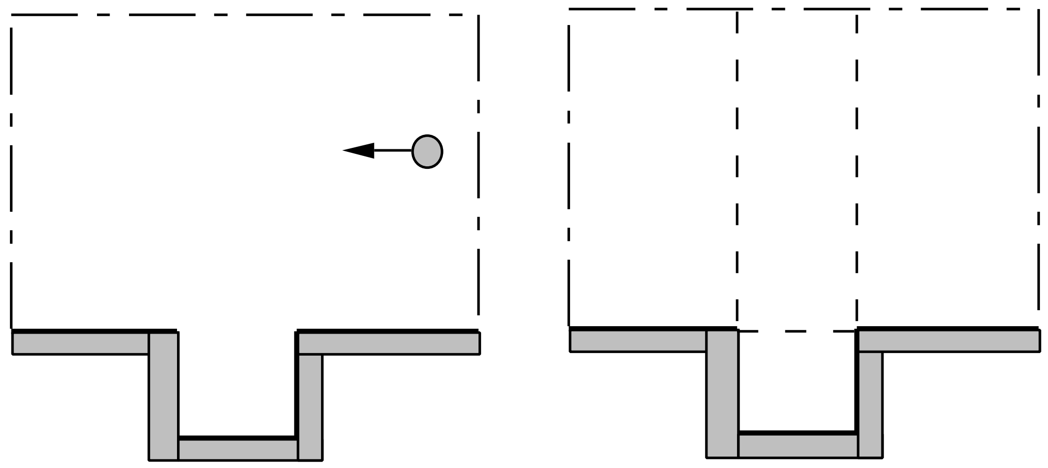
Time
(Allen)

Relation	Symbol	Symbol for Inverse	Pictorial Example
X before Y	<	>	XXX YYY
X equal Y	=	=	XXX YYY
X meets Y	m	mi	XXXYYY
X overlaps Y	o	oi	XXX YYY
X during Y	d	di	XXX YYYYYY
X starts Y	s	si	XXX YYYYY
X finishes Y	f	fi	XXX YYYYY

The thirteenth possible relationships

Space

Space is decomposed in places

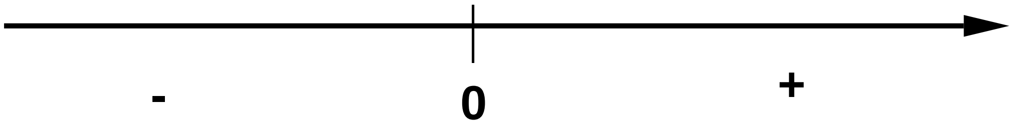


Physical situation **Place vocabulary**

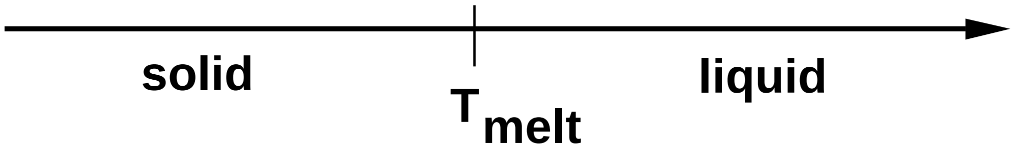
- **Space is decomposed in places that can be reasoned about symbollically.**

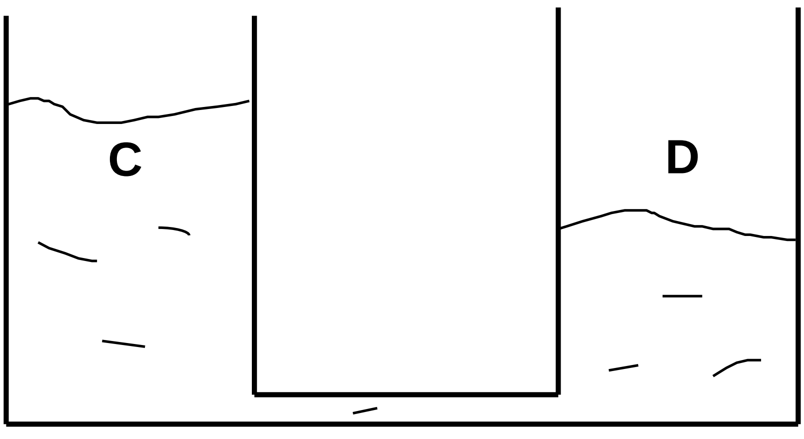
- **An ordering is defined between elements of the place vocabulary.**

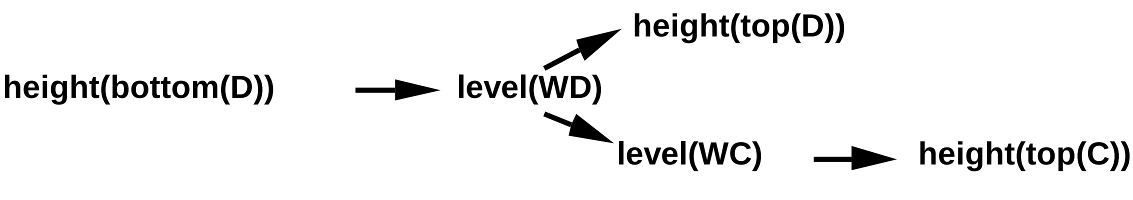
Quantity space



QS = {-,0,+}





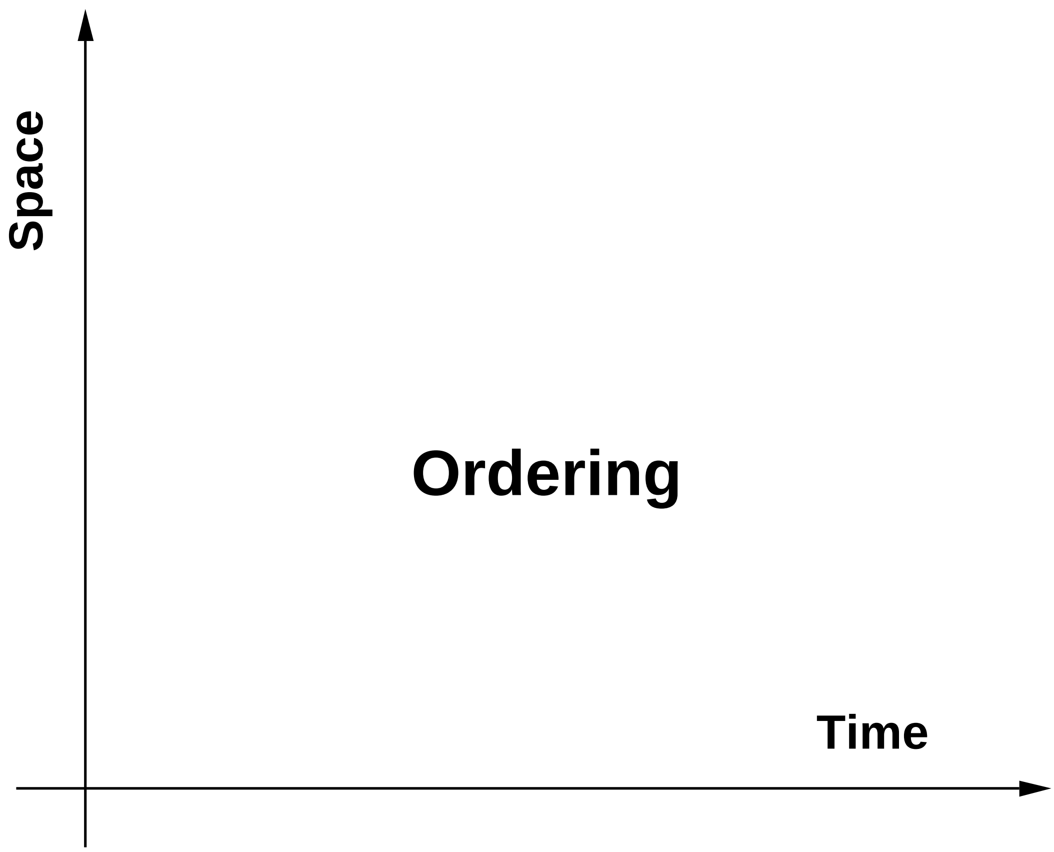


Insight: **Space is defined as a partial ordering between things**

What is time and space?

"Time and space are not things, but orders of things"

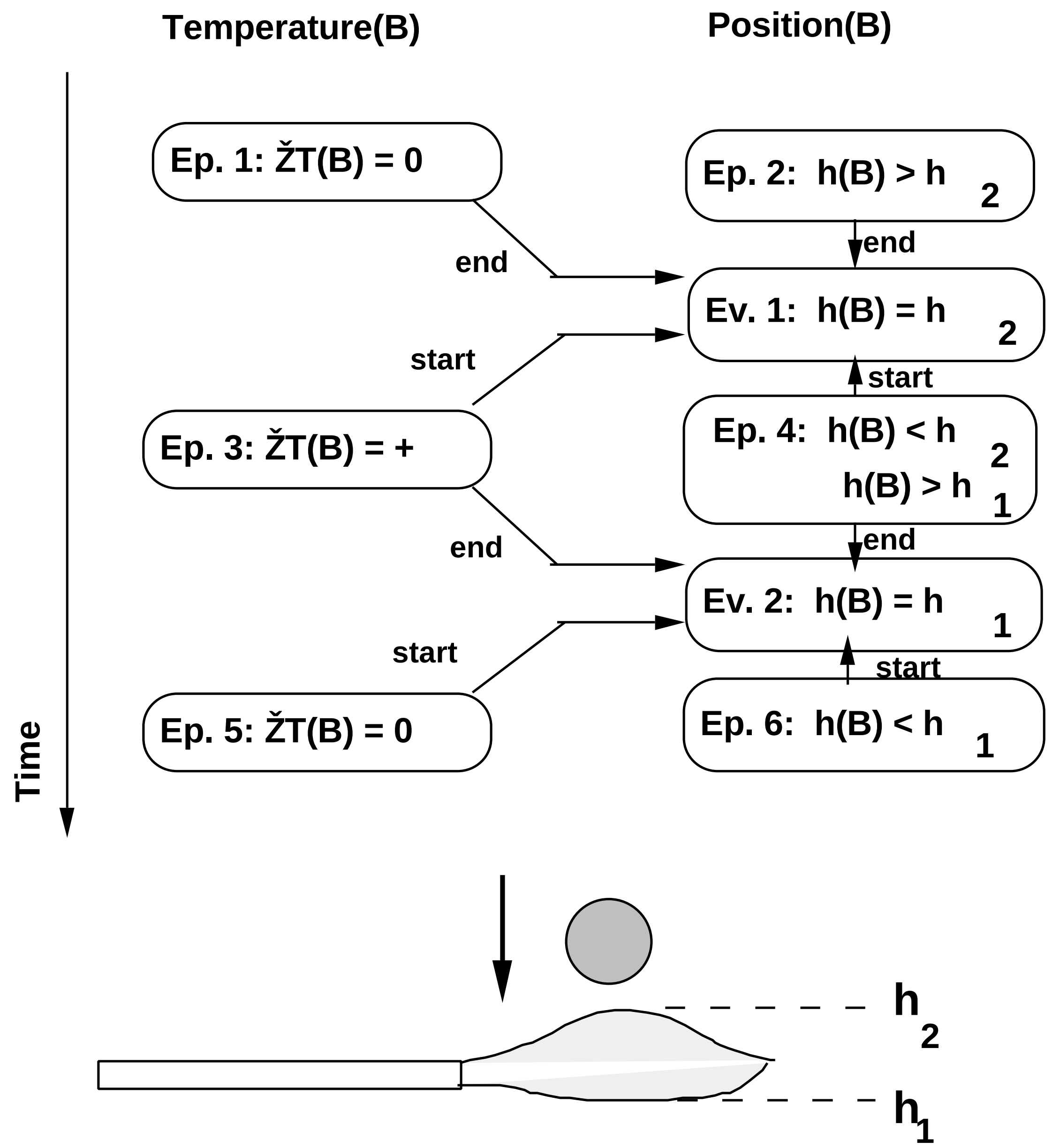
Gottfried Leibnitz



Histories

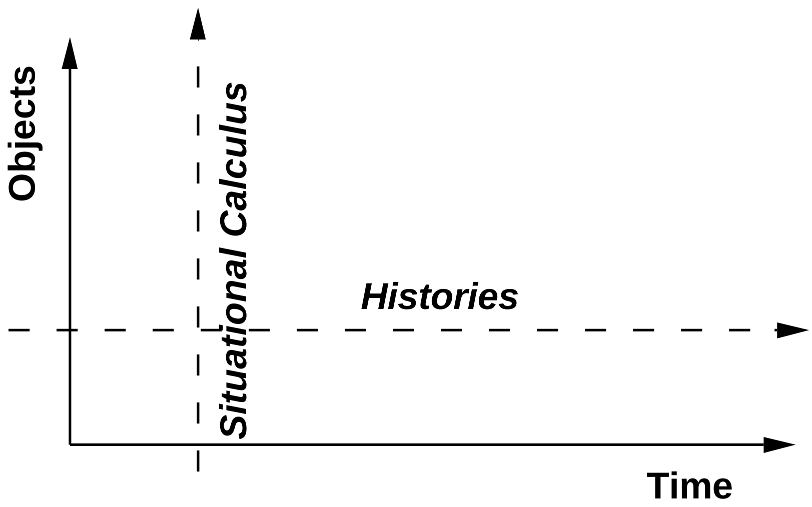
The history of an object is made up of episodes and events which depict the changes concerning that object.

History of B:



Insight: **The history of an object includes the union of its parameter histories.**

Histories versus Situational Calculus



History:	**History of an object over time.**
Situation:	**Description of the world at an instant.**

- **Situational calculus leads to the frame problem: what facts change and what facts do not.**

- **Histories are always spatially bounded.**

- **As a consequence, objects interact only when their histories intersect.**

Where are we?

Kinematics:

What changes occur and how.

Where are we going from here?

Dynamics:

Why do things change?

What causes change?

→ **Causal Reasoning**

Processes Cause Change

(Forbus)

Processes are introduced to make explicit the causes for all change.

Why does the temperature of the steel ball increase?

Because there is a `Heat-Flow` process occurring during the episode when the ball passes through the flame:

```
Process Heat-Flow

Individuals:
  src an object, HasQuantity(src, heat)
  dst an object, HasQuantity(dst, heat)
  path a Heat-Path, Heat-Connection(path,src,dst)

Preconditions:
  Heat-Aligned(Path)

QuantityConditions:
  A[temperature(src)] > A[tenperature(dst)]

Relations:
  Let flow-rate be a quantity
  A[flow-rate] > ZERO
  flow-rate a_Q+ (temperature(src) - temperature(dst))

Influences:
  I^- (heat(src), A[flow-rate])
  I^+ (heat(dst), A[flow-rate])
```

Processes are the only cause for change

- ***Sole mechanism assumption***

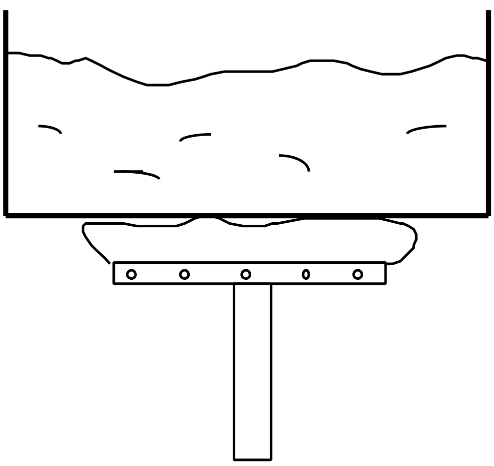
  All changes in physical systems are caused directly or indirectly by processes

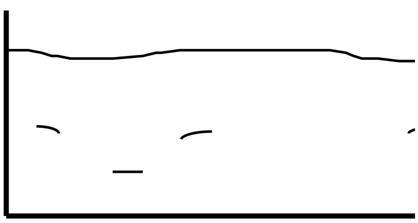
- ***Consequence***

  a) Domain

  b) The dynamics is specified once the list of processes that can occur is explicitely described

  c) If the process vocabulary is complete, then one can reason by exclusion (closed-world assumption).





Processes
for the Steel Ball

Process Heat-Flow

Individuals:
src an object, HasQuantity(src, heat)
dst an object, HasQuantity(dst, heat)
path a Heat-Path, Heat-Connection(path,src,dst)

Preconditions:
Heat-Aligned(Path)

QuantityConditions:
A[temperature(src)] > A[tenperature(dst)]

Relations:
Let flow-rate be a quantity
A[flow-rate] > ZERO
flow-rate a_{Q+}(temperature(src) - temperature(dst))

Influences:
I^{-}(heat(src), A[flow-rate])
I^{+}(heat(dst), A[flow-rate])

Process Motion(B, dir)

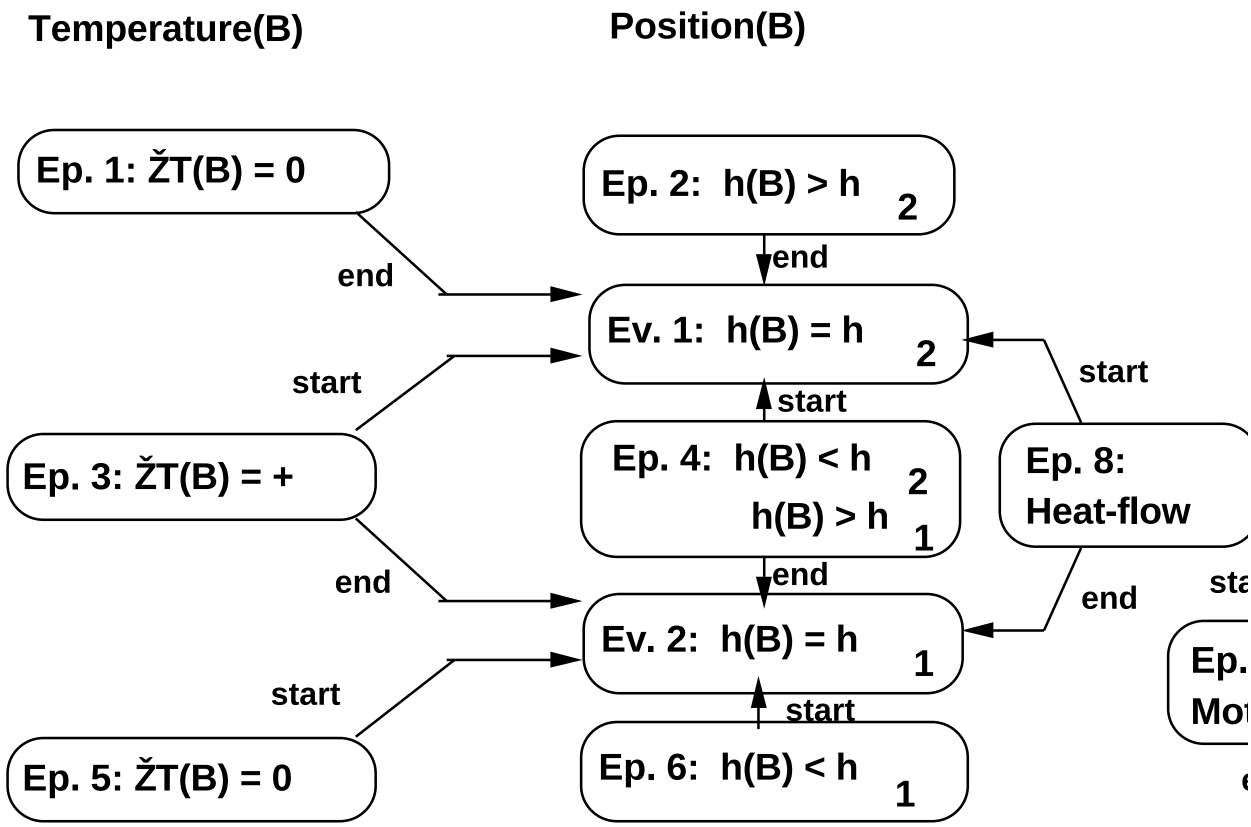
Individuals:
B an object, Mobile(B)
dir a direction

Preconditions:
Free-direction(B,dir)
Direction-Of(dir, velocity(B))

QuantityConditions:
A_m[velocity(B)] > ZERO

Influences:
I^{+}(position(B),A[velocity(B)])

History with Processes



Processes for Impetus

Process Motion

Individuals:
B an object, Mobile(B)
dir a direction

Preconditions:
Free-direction(B,dir)
Direction-Of(dir, impetus(B))

QuantityConditions:
A_m[impetus(B)] > ZERO

Relations:
Let vel be a quantity
vel a_{Q+} impetus(B)

Influences:
I^+(position(B),A[vel])

Process Impart

Individuals:
B an object, Mobile(B)
dir a direction

Preconditions:
Free-direction(B,dir)
Direction-Of(dir, impet

QuantityConditions:
A_m[net-force(B)] > ZERC

Relations:
Let acc be a quantity
acc a_{Q+} net-force(B)
acc a_{Q-} mass(B)

Influences:
I^+(impetus(B),A[acc])

Process Dissipate

Individuals:
B an object, Mobile(B)

QuantityConditions:
A_m[impetus(B)] > ZERO

Relations:
Let acc be a quantity
A_s[acc] = A_s [impetus(B)]

Influences:
I^-(impetus(B),A[acc])

IQ Analysis:
Discovering Causality & Function

(De Kleer)

IQ Analysis
(De Kleer)

- Causal propagation

- Causal explanation

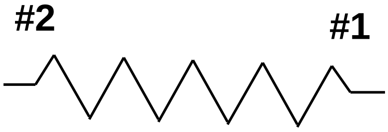
- Understanding feedback

- Mythical time

Causal Analysis

GIVEN

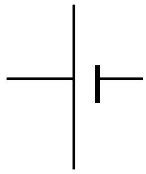
A) Components ® Behavioral laws



Resistor

$[v_{\#2,\#1}] = [i_{\#2,\#1}]$

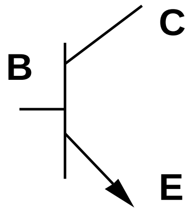
Ž$v_{\#2,\#1}$ = Ž$i_{\#2,\#1}$



Battery

$[v] = +$

Ž$^{n}v = 0$



Transistor

Ž$v_{B,E}$ Þ Ži_{C}

Ž$v_{B,E}$ - Þ Ži_{E}

Ž$v_{B,E}$ Þ Ži_{B}

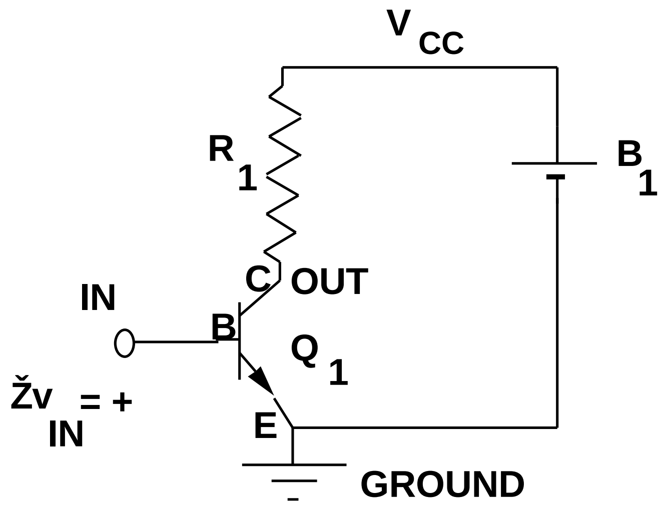
B) Structure: Topology of the device

C) Perturbation of a quiescent state

[Žv_{IN}] = +

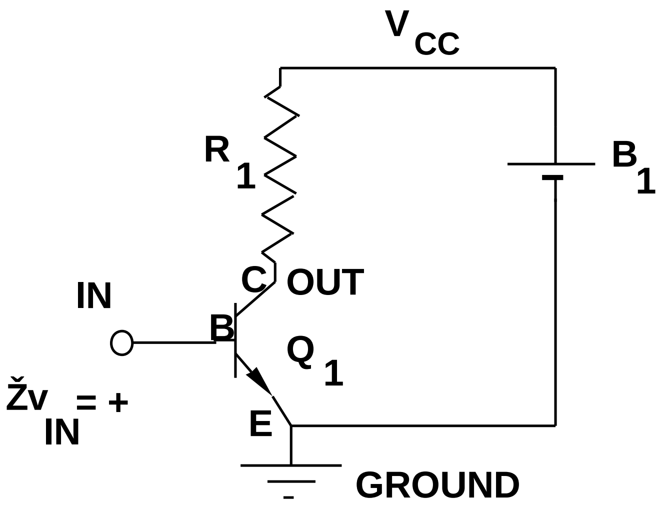
Produces

- The response of the device
- A causal account for the incremental change

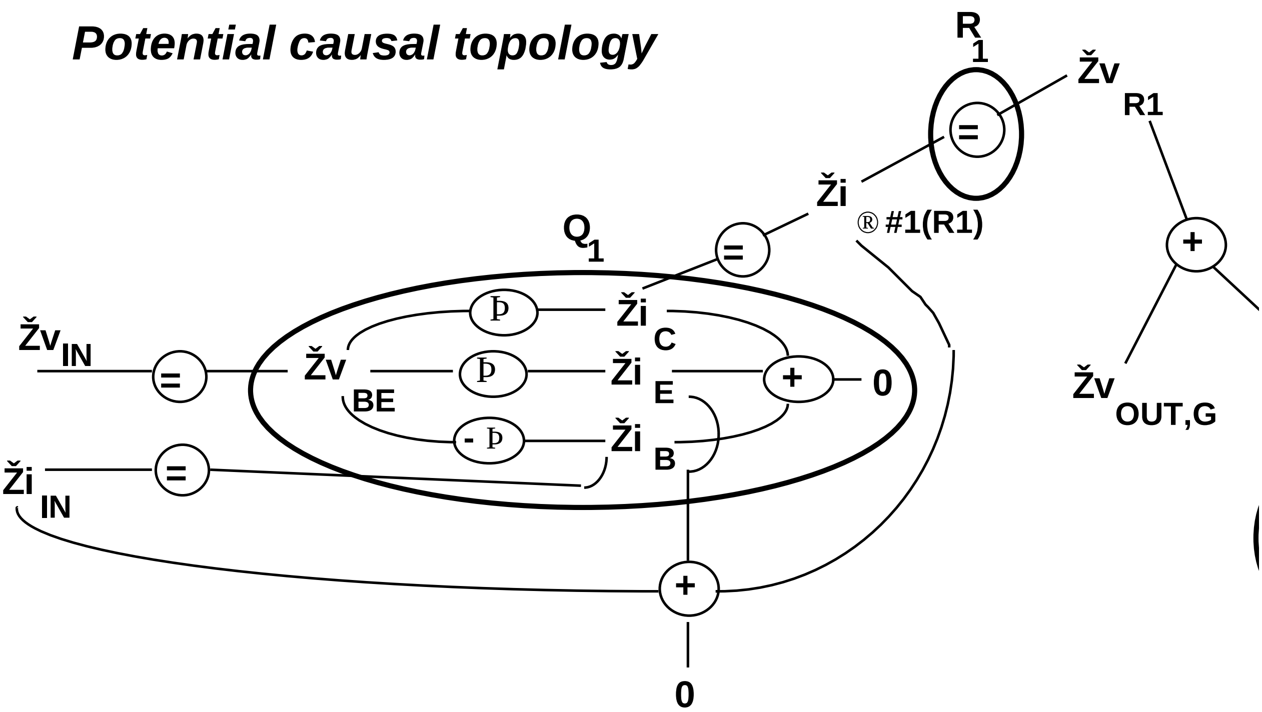


Antecedents		Event	Reason
		$Žv_{IN}$ = +	Given
	Þ	$Ži_{® C(Q1)}$ = +	$Žv_{B,E}$ Þ $Ži_{C}$ f
	Þ	$Ži_{¬ \#2(R1)}$ = +	KCL for node O
	Þ	$Ži_{® \#1(R1)}$ = +	KCL for res. R
	Þ	$Žv_{CC,OUT}$ = +	$Žv_{\#2,\#1}$ = $Ži_{\#}$ for resistor R
		$Žv_{CC}$ = 0	$Žv_{+,-}$ = 0 for battery B_{1}
$Žv_{CC}$ = 0, $Žv_{CC,OUT}$ = +	Þ	$Žv_{OUT}$ = -	KVL applied to nodes OUT, VC GROUND

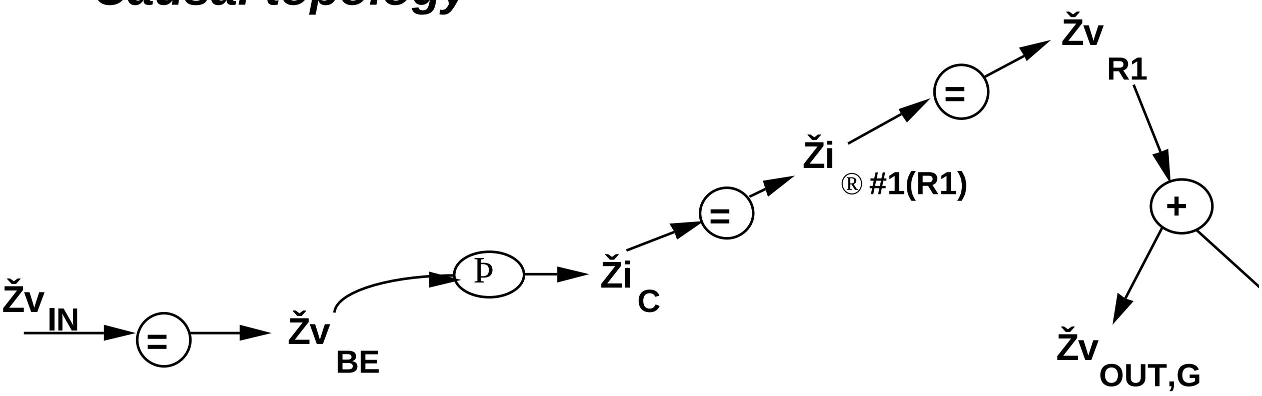
Local Propagation



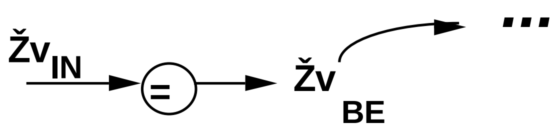
Potential causal topology



Causal topology



Mythical Time



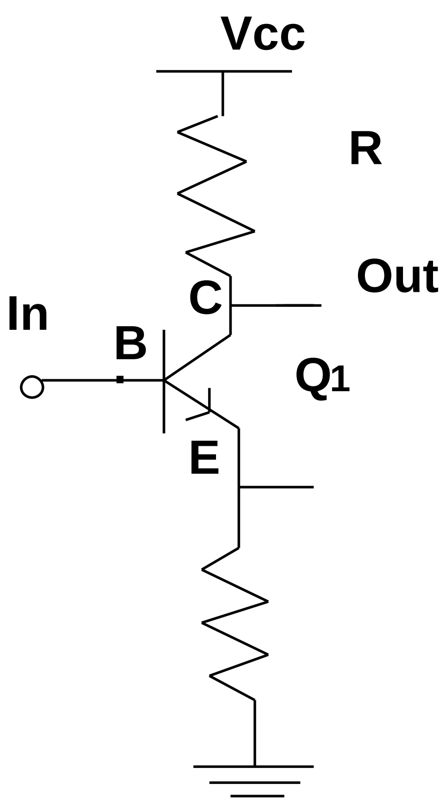
Cause ® Effect

No elapsed time between different steps of the causal propagation.

Ordering ® Mythical

Causal Heuristics

- Causal Propagation gets stuck



Given:

$\check{Z}v_{IN,G} = +$

Impass

$\check{Z}v_{IN,G} - \check{Z}v_{IN,E} + \check{Z}v_{E,G} -$

E.g: Amplifier Input

Insight: "Causal pertubation path has not yet attained the emmiter"

KVL Heuristic
(= Component Heuristic)

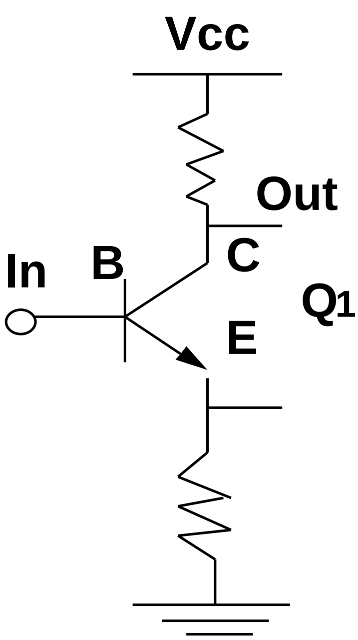
- **Impass**

  Žv$_{n,G}$ = ± (G, ground)

  Žv$_{n,G}$ - Žv$_{n,m}$ - Žv$_{m,G}$ - 0 (KVL)

- **Assume**

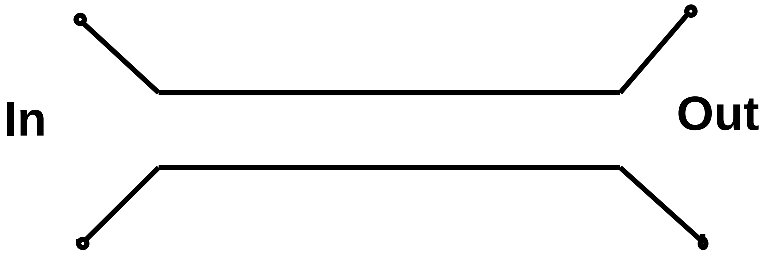
Žv$_{n,G}$ 📪 Žv$_{n,m}$



	Effect	**Reason**
	Žv$_{IN,G}$ = +	Given
📪	Žv$_{IN,E}$ = +	KVL Heurist for Transistor Q
📪	Ži ® C(Q 1) = +	Žv$_{B,E}$ 📪 Ž Transistor Q

Component Heuristic

- For fluids



Pipe

ŽP$_{IN}$ - ŽP$_{IN,OUT}$ - ŽP$_{OUT}$ - 0

ŽQ$_{IN,OUT}$ - ŽP$_{IN,OUT}$ - 0

ŽP$_{IN}$ = +

- IMPASS

-

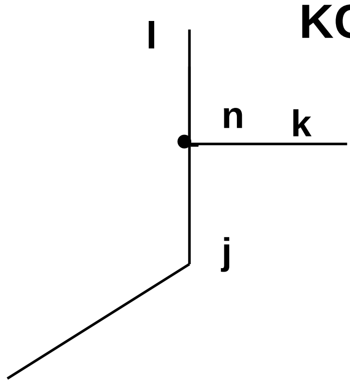
Effect	Reason
ŽP$_{IN,OUT}$ = +	ŽP$_{IN}$ ŽP$_{IN,OUT}$ Component heuristic
ŽQ$_{IN,OUT}$ = +	ŽP$_{IN,OUT}$ ŽQ$_{IN,OUT}$ Pipe

Intuition: **The pertubation has not yet reached node out.**

KCL Heuristic
(= Conduit Heuristic)

- **Rule:**

Impass | **Node** | **KCL**



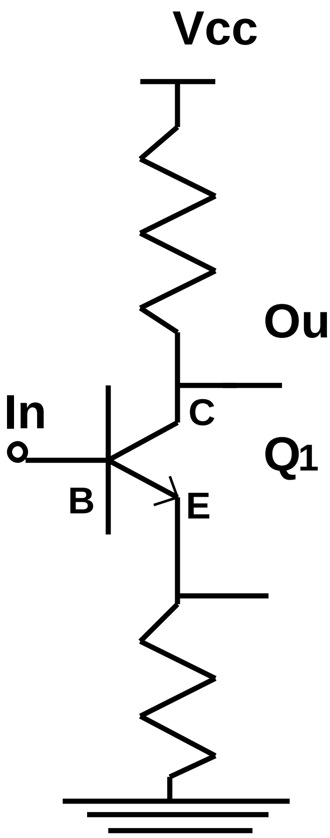
$Ži_{jn} + Ži_{kn} + Ži_{ln} = 0$

$Ži_{jn} = \pm$

- **Assume**

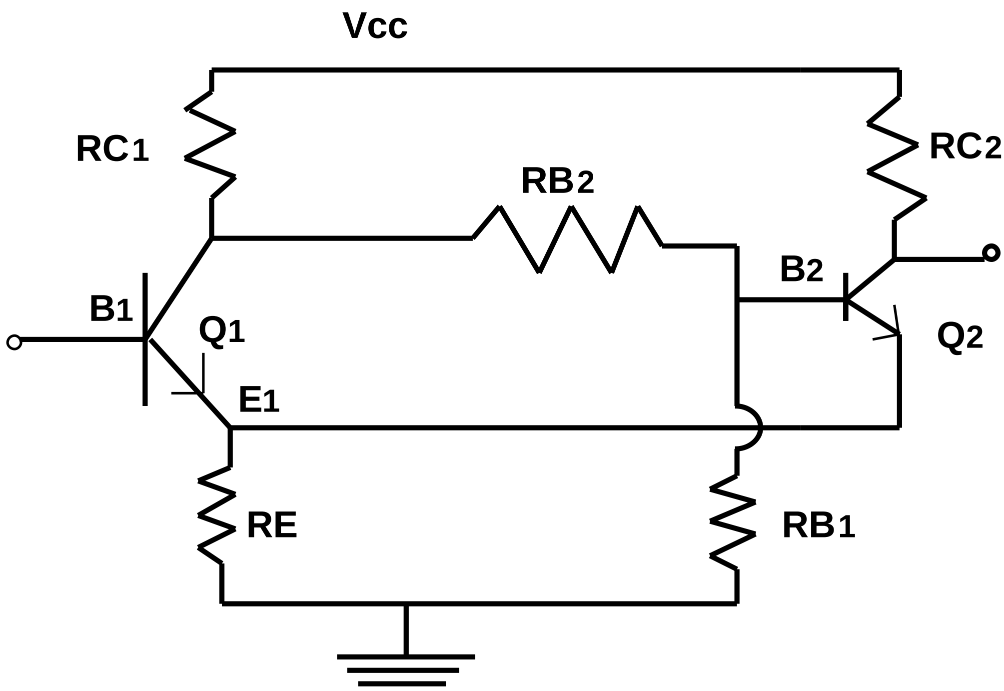
$Ži_{jn}$	📪	$Žv_{n,G}$

- **E.g:**



Effect		Reason
	$Žv_{in} = +$	Given
📪	$Žv_{B,E} = +$	KVL heuristic
📪	$Ži$ ® C(Q1) = +	Transistor Mo
📪	$Žv_{OUT} = -$	KCL heuristic for node OUT

Schmitt trigger



	Effect		Reason
	Žv$_{B1}$ = +		Given
Þ	Žv $_{B1,E1}$ = +	Þ	KVL heuristic for Q1
Þ	Ži® $_{C(Q1)}$ = +		Žv $_{B,E}$ Þ Ži $_{C}$ for transistor Q1
Þ	Žv$_{C1}$ = -		KCL heuristic for node C1
Þ	Žv$_{C1,B2}$ = -		KVL heuristic for resistor RB2
Þ	Ži® $_{\#1(RB2)}$ = -		Resistor RB2
Þ	Ži® $_{\#2(RB2)}$ = +		KCL for resistor RB2
Þ	Žv$_{B2}$ = -		KCL heuristic for node B2
Þ	Žv$_{B2,E1}$ = -		KVL heuristic transistor Q2
Þ	Ži ¬ $_{E(Q2)}$ = -		Žv $_{B,E}$ Þ Ži $_{E}$ for transistor Q2
Þ	Žv $_{E1}$ = -		KCL heuristic for node E1

Reason Maintenance Systems

- Heuristic → Guessing
- Encoded → Assumption
- Causal Analysis → Hypothetical Reasoning

- E.g: Žx + Žy - Žz - 0

Assumption	Justification	Consequent
{Žx = +}		Žx = +
{Žy = +}		Žx
{Žx = +, Žy = +}	Žx = + Žy = + ® Žz = +	Žz = +

1) Basic Principles:

a) Forward chaining

b) Propagate assumption with consequent

2) Advantage:

if we already know that Žz = + ® $\perp$

then assumption to reconsider

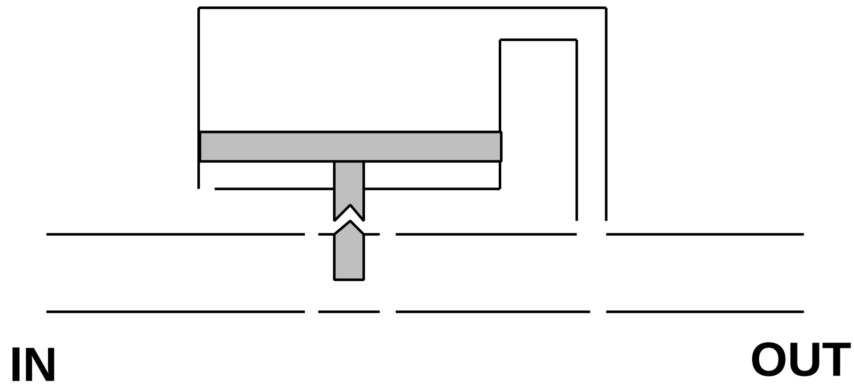
Feedback

- **Cause(A)** Þ Å Þ **Effect(C)**

  Ý ß

  feedback(B)

- **Example: Pressure Regulator**



ŽP$_{IN}$ = +

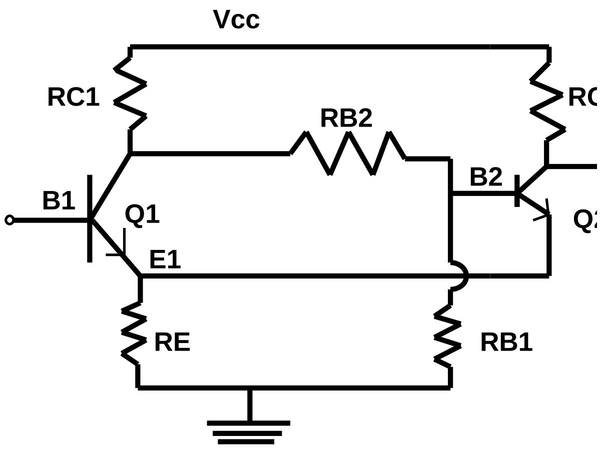
Þ **ŽP$_{IN,OUT}$ = +**

Þ **ŽQ$_{IN,OUT}$ = +**

Þ **ŽP$_{OUT}$ = +**

Mythical Time and Feedback

- **Causal Propagation** *(Schmitt triger)*



Žv$_{B1}$ **= +**

Žv$_{B1,E1}$ **= +**

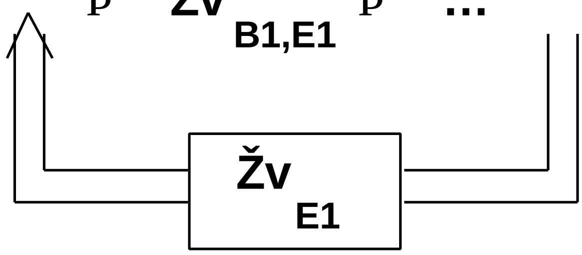
•

•

•

Žv$_{E1}$ **= -**

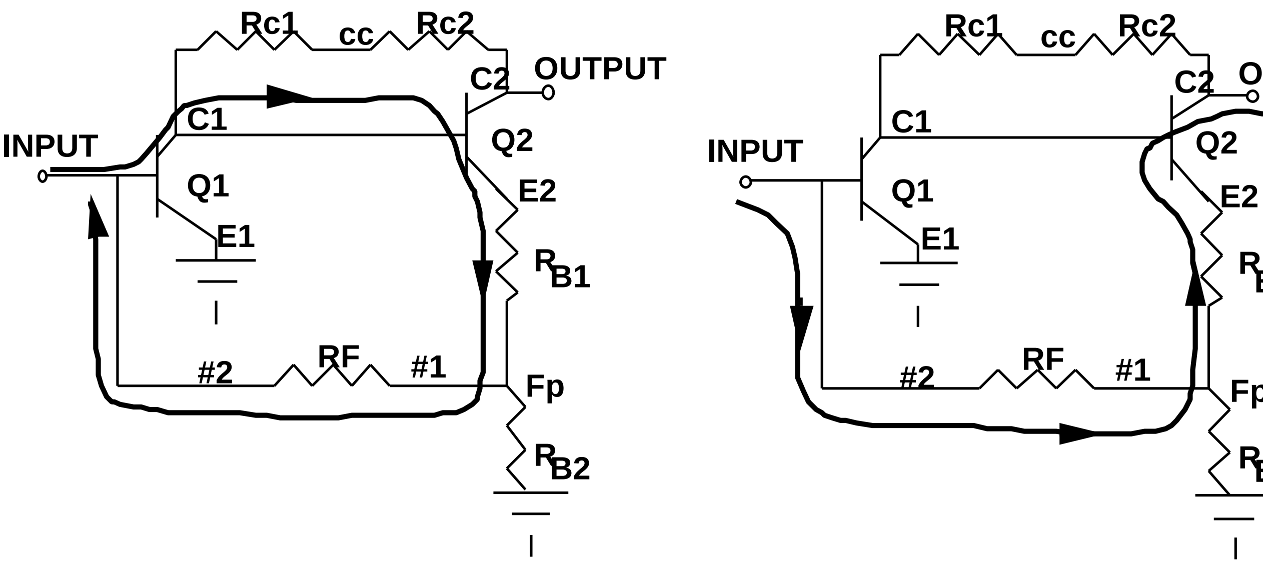
Žv$_{B1}$ Þ Þ **Žv**$_{B1,E1}$ Þ **…**



- **Žv**$_{B1,E1}$ **- - Žv**$_{E1}$ **+ Žv**$_{B1}$
- **Žv**$_{E1}$ **= - f (Žv**$_{B1,E1}$**)**
- **Thus** **Žv**$_{B1,E1}$ **- + f (Žv**$_{B1,E1}$**) + Žv**$_{B1}$
- **Feedback positif**

Multiple Interpretations

• Several interpretations



• Completeness

• Realizability

• Intended interpretation ® Function

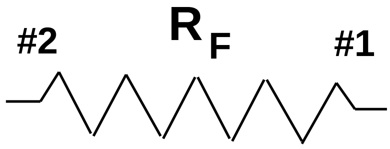
Insight: **Ambiguities in causal analysis can be removed by using other sources of knowledge**

Teleological Analysis

- ***What is the role of resistor RF in "feedback amplifier"?***

- **Library of primitive teleological fragments**

<u>Example</u> : Resistor



	output: $i_{\#1}$	output: $i_{\#2}$
input: $i_{\#1}$	*	bias
input: $v_{\#1}$	v-load	v-to-i-couple
input: $v_{\#1,\#2}$	v-sensor	v-sensor

Causal pattern | **Teleological fragmen**

Žv #1,#2 = - | **input v #1,#2**

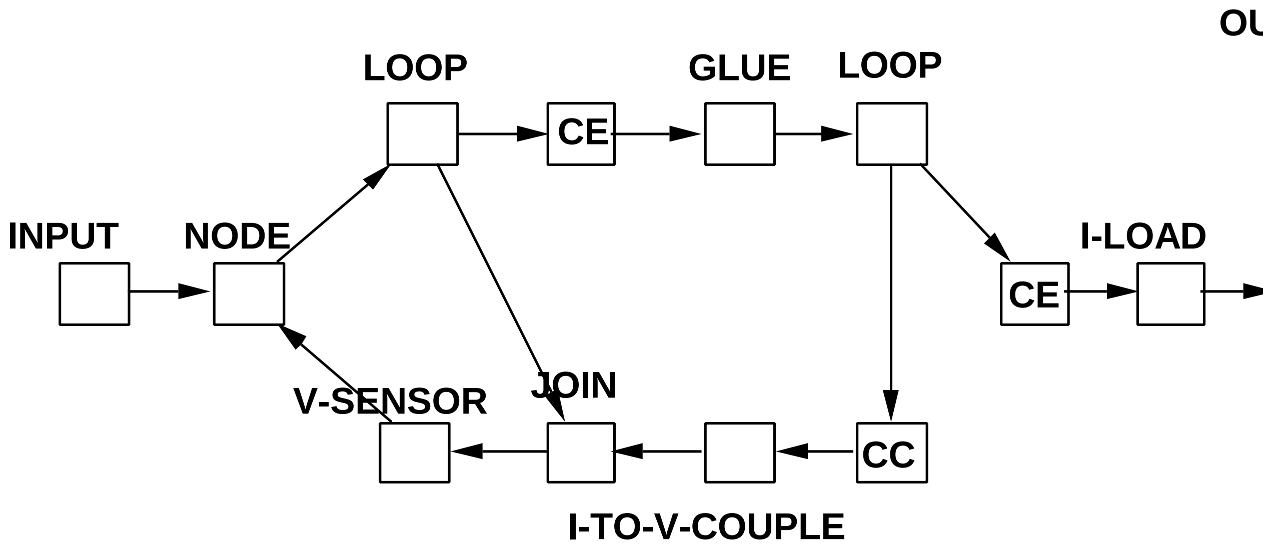
Þ Ži ¬ #1 = - | **output i #1**

R_F functions as voltage sensor

Insight: **In a well-designed artefact, each component has been introduced to fulfill a given purpose**

Taxonomy of Component purposes

Example: Role of each component in CE-feedback



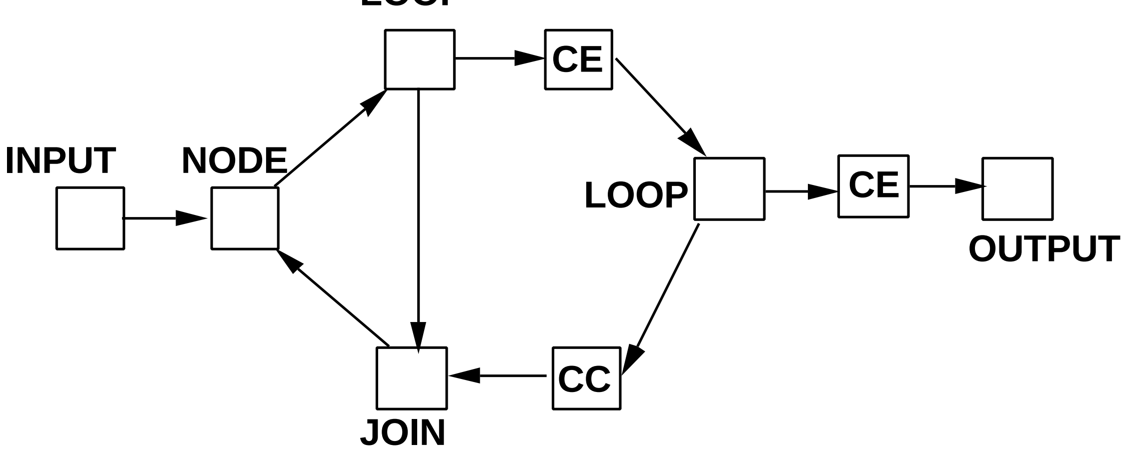
Abstraction

What is the ultimate purpose of CE-feedback?

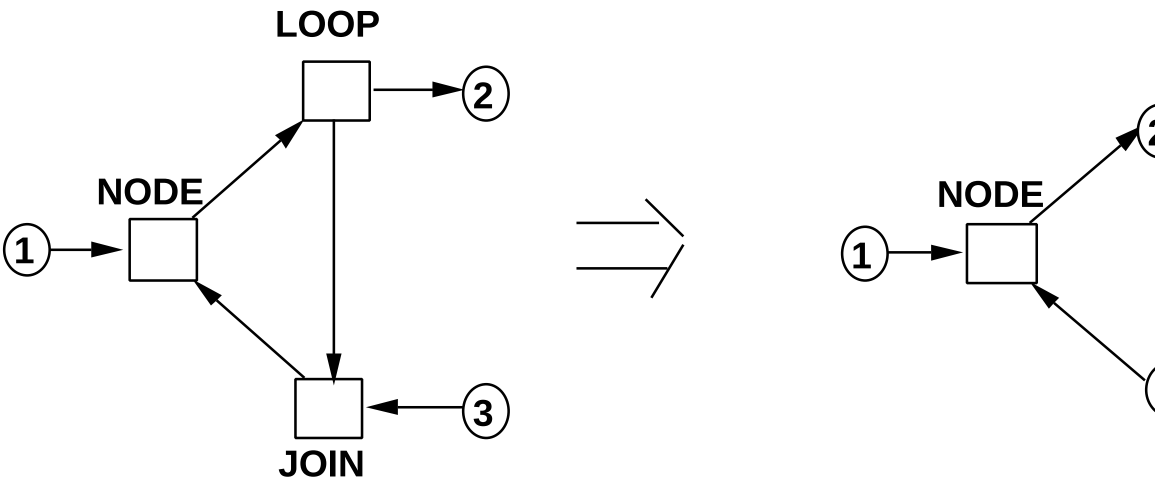
1) Bottom-up passing

2) Class table

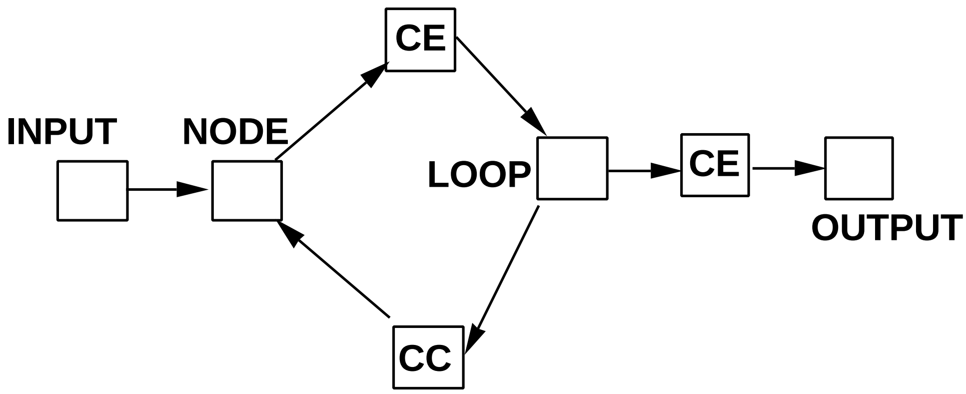
Class	Type	Description
IO	INPUT,OUTPUT	signals on boundar
SAMPLING	NODE,LOOP	feedback sampling p
COMPARISON	NODE,LOOP	feedback comparison
SPLIT	unused	signal splits n ways
JOIN	VOLTAGES,CURRENTS	n signals combine
STAGE	CE,CC,CB,CASCADE,FEEDBACK	amplifying stage
DIFF-2-1	SUM	differential amplifier
COUPLING	GLUE,V-LOAD,WIRE,I-LOAD, V-SENSOR,V-TO-I-COUPLING, I-TO-V-COUPLING,SENSE-Q,LOAD-Q, COUPLE-Q,BIAS,LEVEL	signal couplers



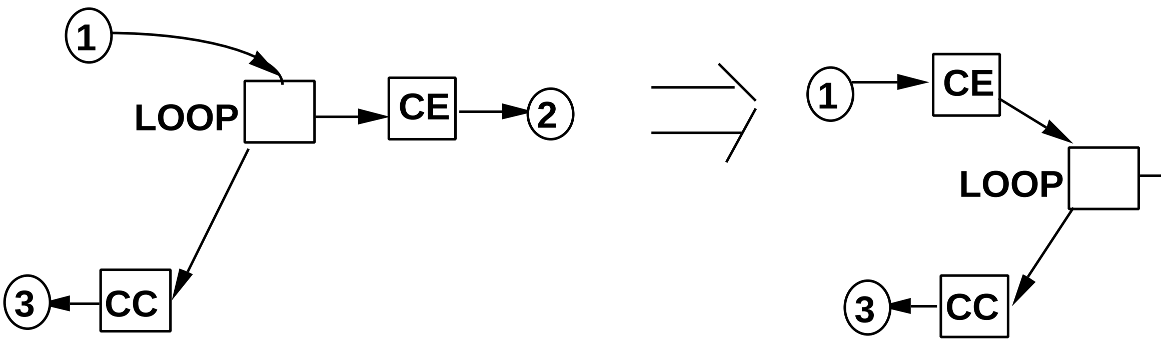
Parse of CE-feedback after removing COUPLING



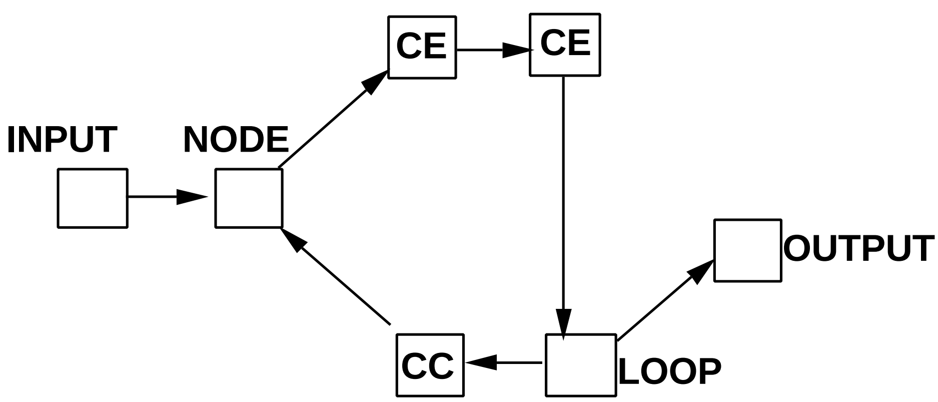
Local feedback substitution rule

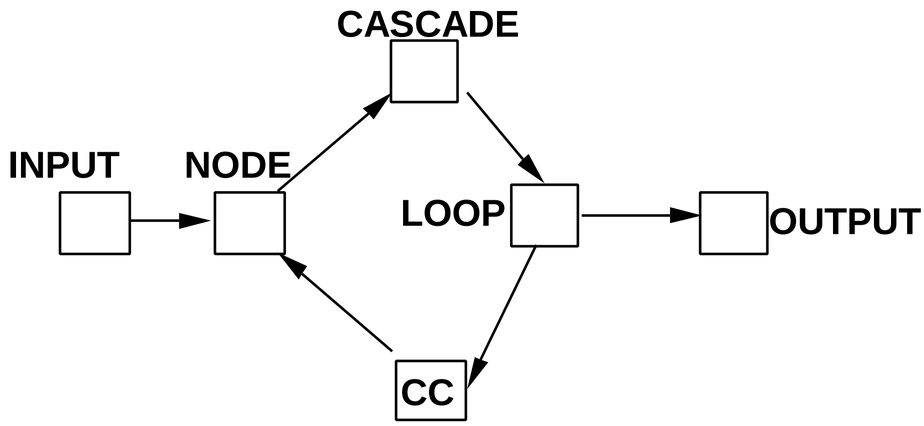


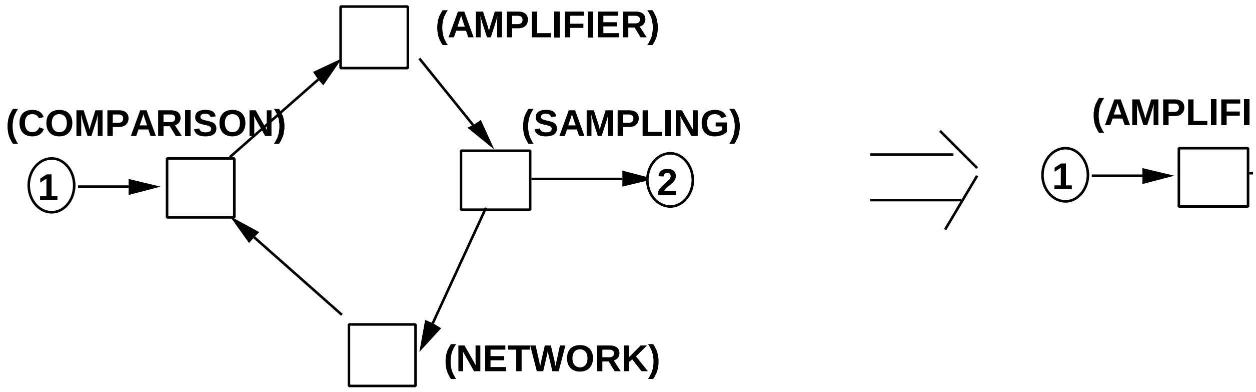
Parse of CE-feedback after removing local feedback



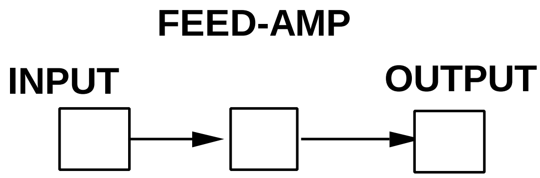
Sampling rewrite rule







Feedback rewrite rule



Parse of CE-feedback
after eliminating feedback

Teleology Disambiguates Causal Analysis

- **Example: Several causal analysis for the CE-feedback**

- **Teleology allows to define preference**

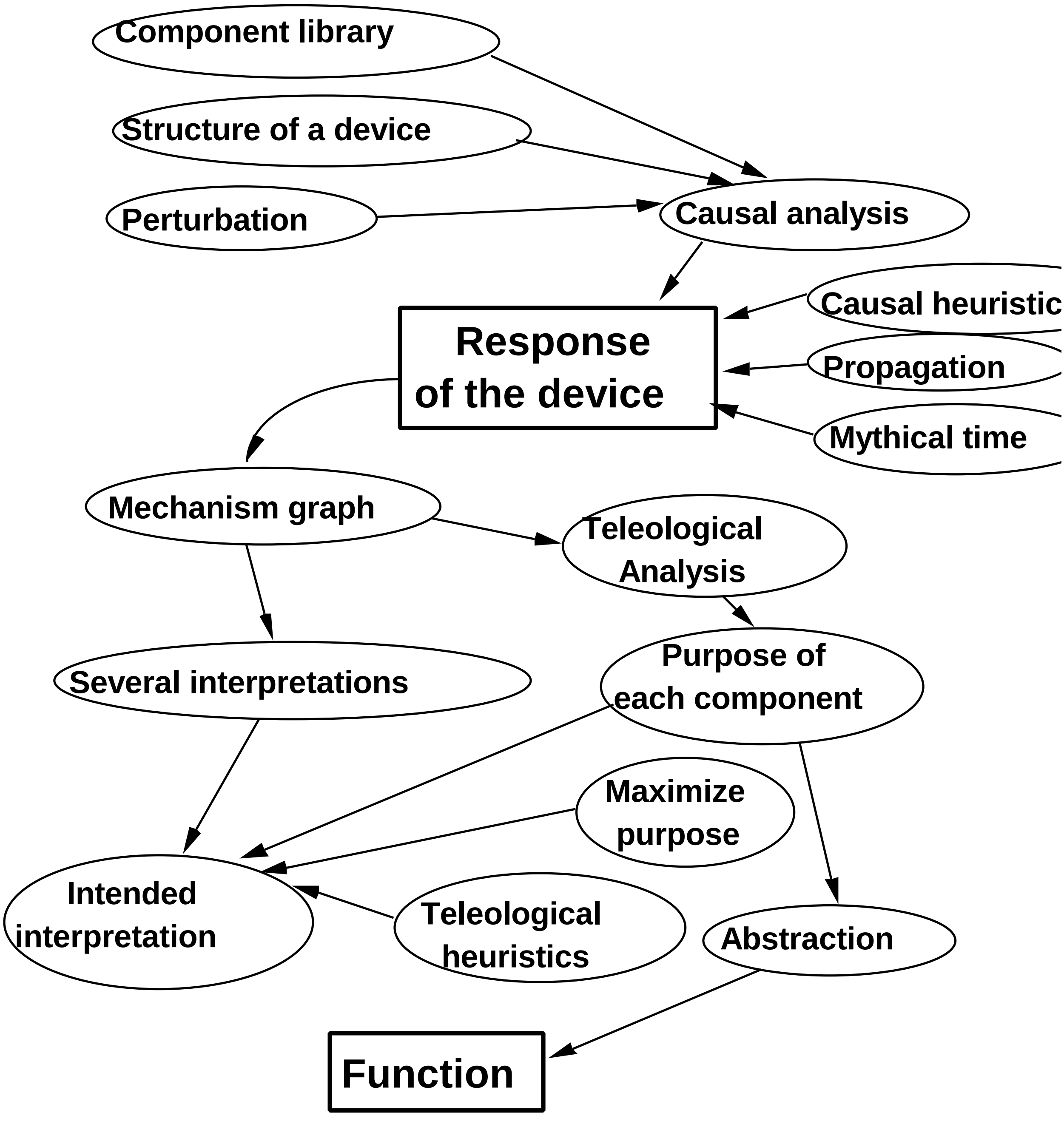
  - **Maximize purpose (set inclusion)**

  - **Rule out implausible purposes**

  - **Preference rule**

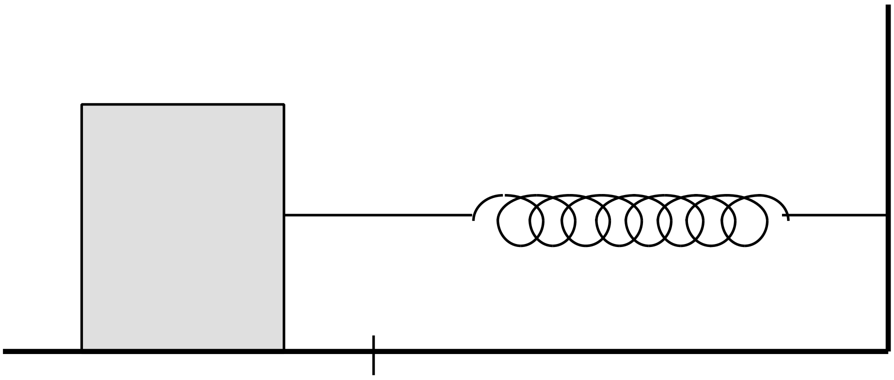
Insight: **Knowing that each component has a purpose, although this purpose must be found, allows to select the intended interpretation and to disambiguate causal analysis**

IQ Analysis Summary

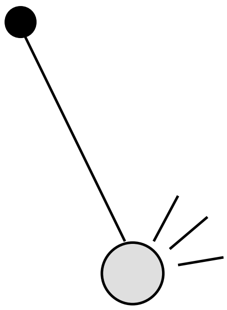


Envisioning

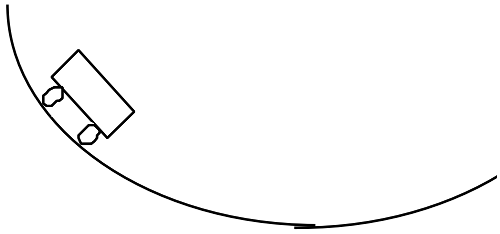
Oscillators



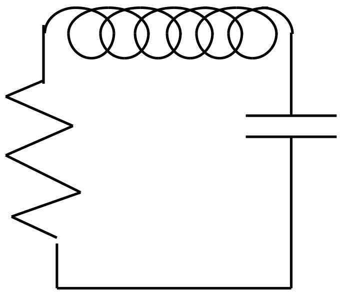
A block and a spring



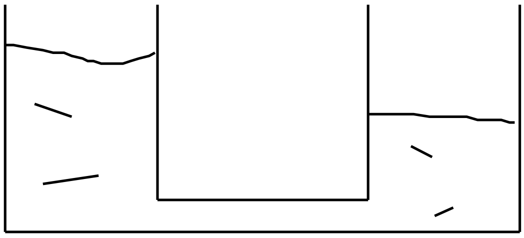
A pendulum



A roller-coaster



A RLC circuit



Notice: **All these oscillators have the same *linear* qualitative model**

Model

Frictionless oscillator

$$F = -kx$$

$$F = m\ddot{x}$$

$$Ž^2 x + x - 0$$

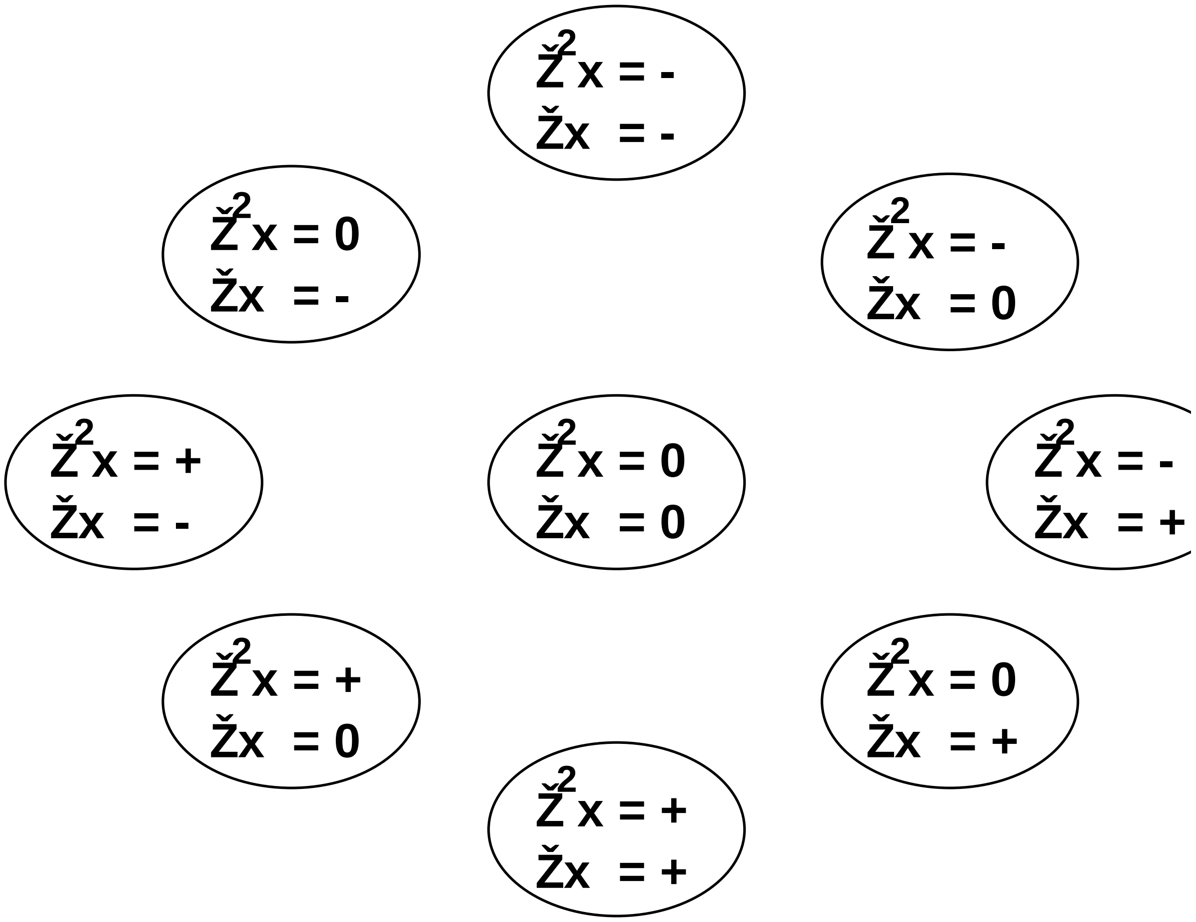
Damped oscillator

$$F = -kx - f\dot{x}$$

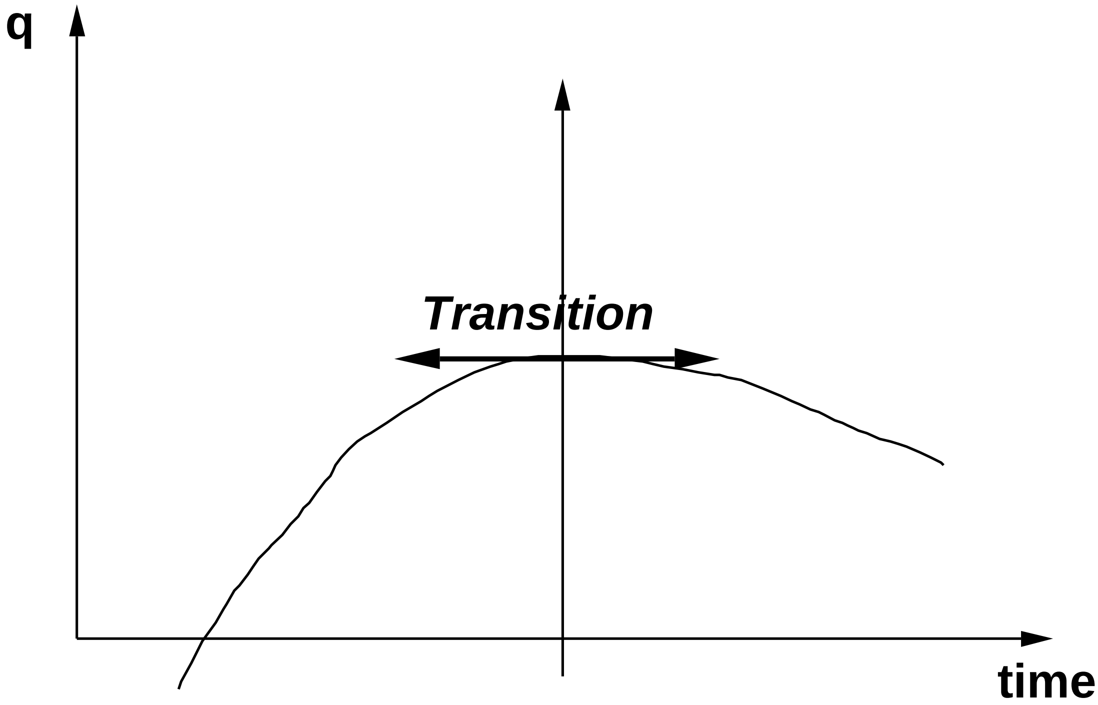
$$F = m\ddot{x}$$

$$Ž^2 x + Žx + x - 0$$

States


Ž2x = -
Žx = -
Ž2x = 0
Žx = -
Ž2x = -
Žx = 0
Ž2x = +
Žx = -
Ž2x = 0
Žx = 0
Ž2x = -
Žx = +
Ž2x = +
Žx = 0
Ž2x = 0
Žx = +
Ž2x = +
Žx = +

Performing
Transition Analysis



Transition analysis:

- **Finding next event**

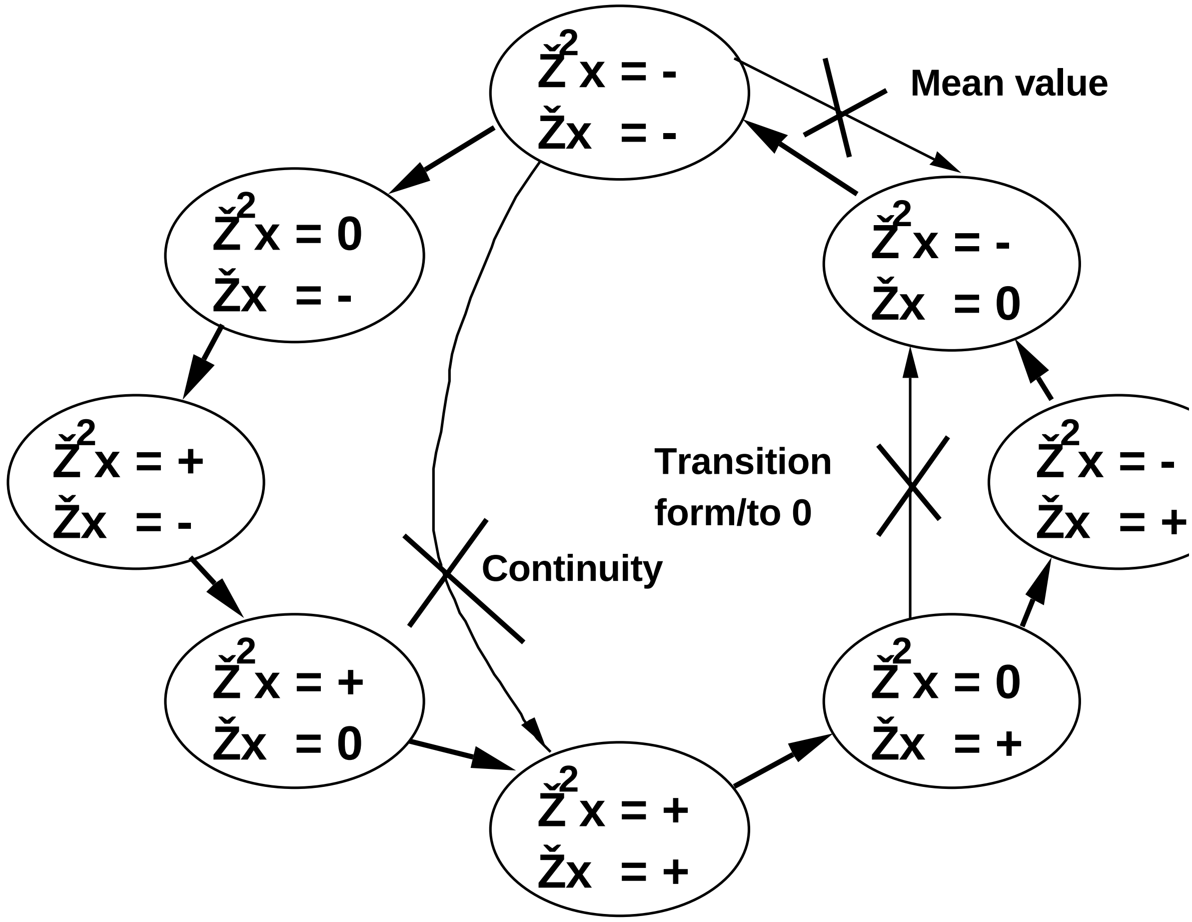
- **Change in qualitative region**

x = +		**x = +**
Žx = +	→	**Žx = 0**
Ž²x = -		**Ž²x = 0**

- **Change in operation region**

Transistor, pressure regulator

Transition Recognition



- **Continuity: q = + ® q = -**

- **Mean value q(t2) - q(t1) + Žq(t1)**

- **Transitions from/to 0:**

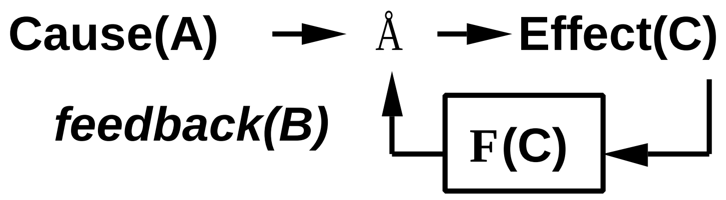
  Transitions to 0 take time
  Transitions from 0 are instantaneous

<u>Thus</u> :

  Transition to 0 and transition from 0 cannot happen simultaneously

Resistive Feedback

A + B = C **B = F(C)**



If [A] = [B]
Thus [C] = [A]

If [B] = -[A]

The feedback parameter B reduces the effect of the input A on C

F resistive

{ **F polynomial**
No memory

Delay
(Williaws)

E.g. { [B] = - [C]
A + B = C
Negative feedback

Interpretation:

1) [A] = + → [C] = +

No change yet on B *(Mythical time)*

2) Then [C] = + → [B] = -

3) If |B| = |A| then { [C] = 0
[C] = - [B] Þ [B] = 0

Part 2

Enhancing
Qualitative Simulation

Comparative Analysis

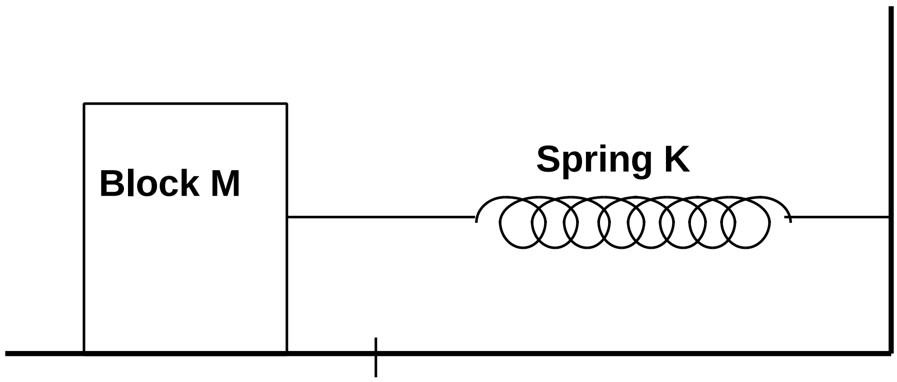
Comparative Analysis

(Weld)

- ***Qualitative Simulation:***

  Structure ® Behavior

- ***Example:***

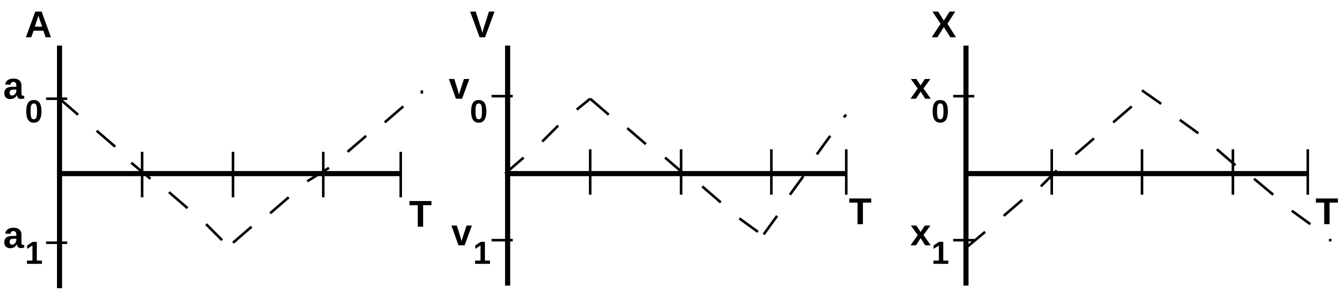


F = MA **F = -KX**

M > 0 **-K < 0**

V(0) < 0 **X(0) = x_0 < 0**

- ***QSIM behavior:***



Comparative Analysis 2

- ***Comparative Analysis:***

Behavior + Perturbation	®	How and why the behavior changes

- ***Example:***

  What happens if the mass of the block is increased?

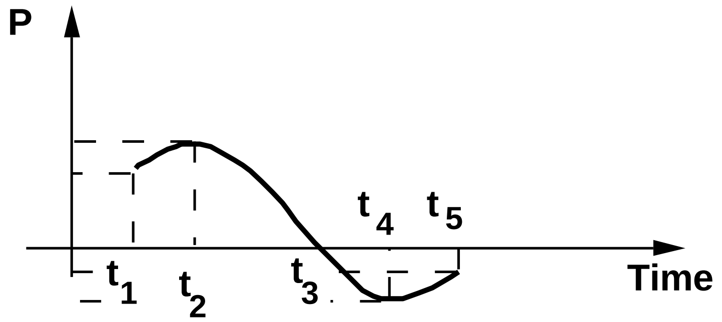
- ***Answer:***

  "Since force is inversely proportional to position, the force on the block will remain the same when the mass is increased. But if the block is heavier, then it won't accelerate as fast. And if it doesn't accelerate as fast, then it will always be going slower and so will take longer to complete a full period (assuming it travels the same distance)."

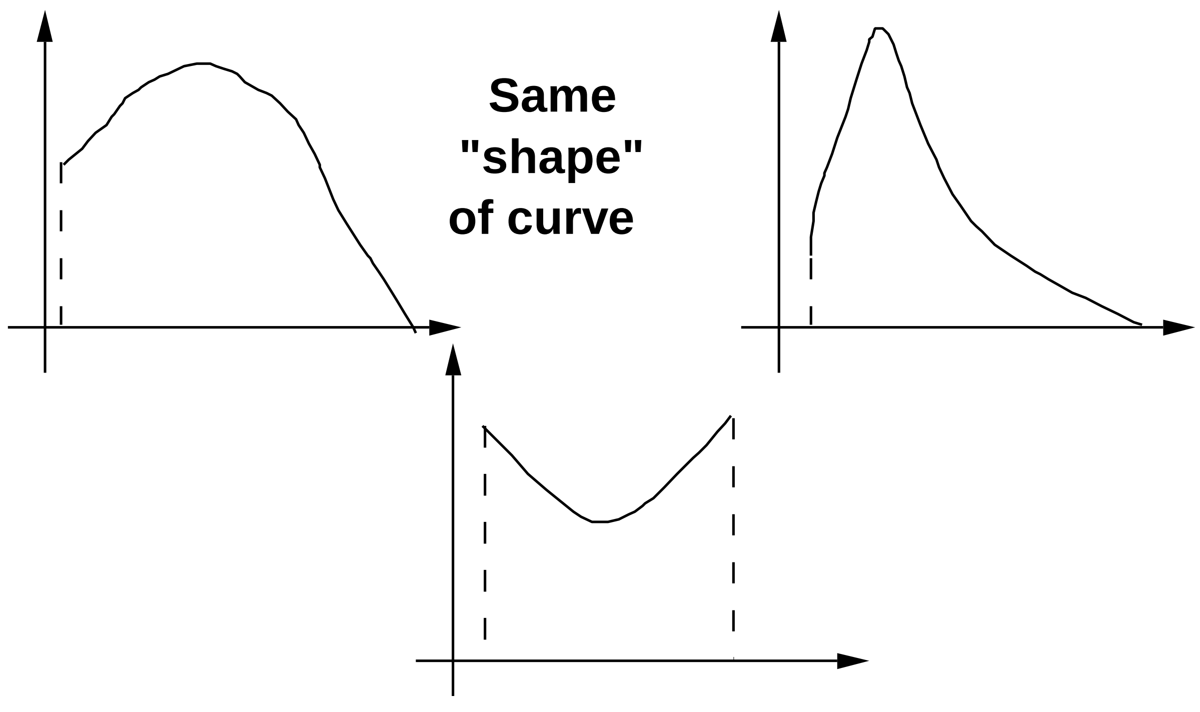
Qualitative Behaviors

Qualitative Behavior:

- **0 < … < k** — **Transitions**
- **t_i** — **Time when transition i occurs**
- **$p_i = P(t_i)$** — **Landmark for P at transition i**



Topologically equal qualitative behaviors:

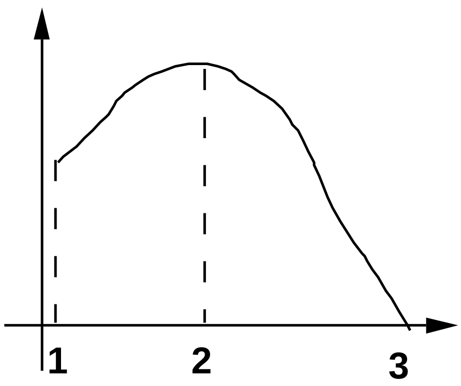


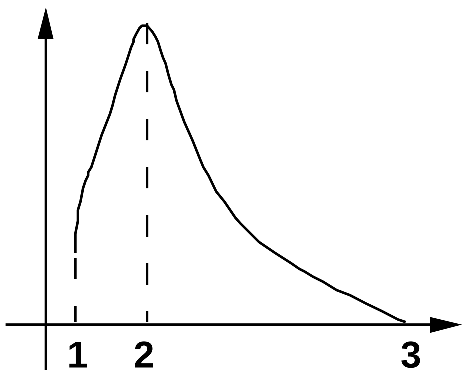
Relative Change

PÝ$_i$ if $\hat{p}_i > p_i$

P ||$_i$ if $\hat{p}_i = p_i$

Pß$_i$ if $\hat{p}_i < p_i$

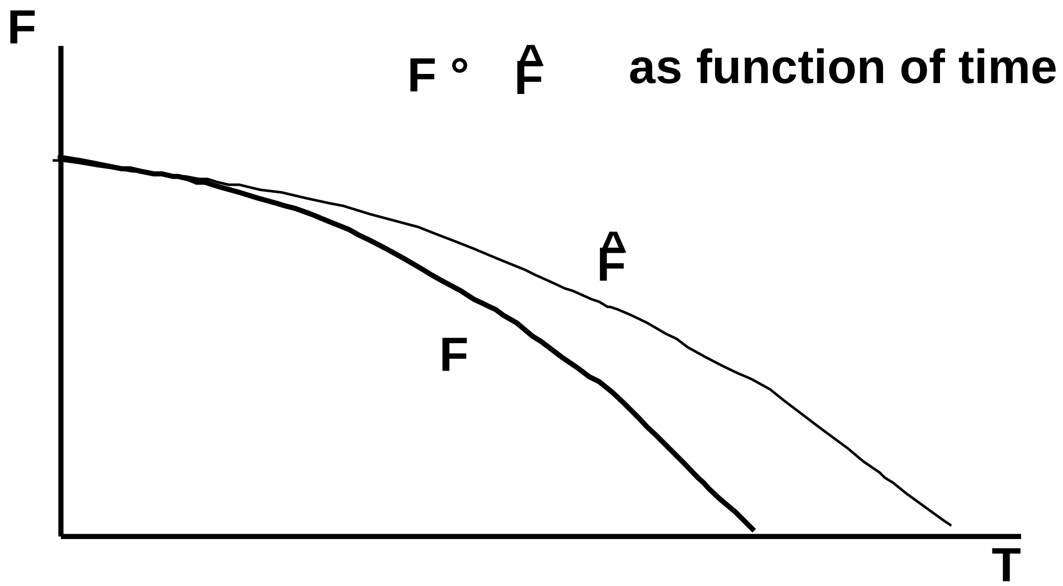


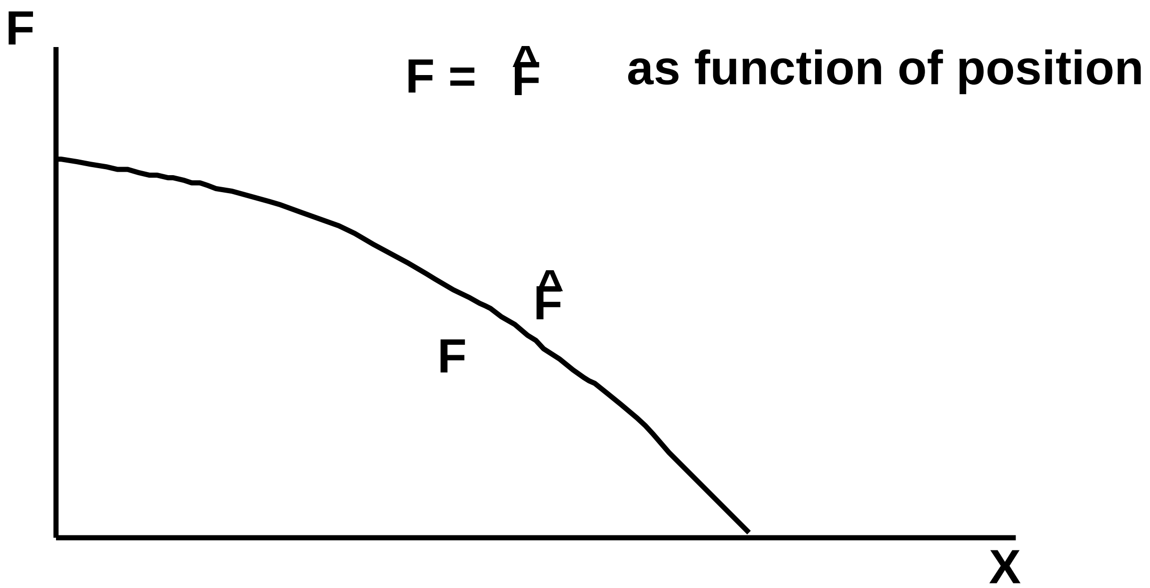


PÝ$_1$ P ||$_2$ Pß$_3$

Ambiguity

"… the force on the block will remain the same when the mass is increased…"





Reparametrization

Perspectives

Definition

A new reference parameter X is a *covering perspective* over (i,i+1) when:

1) ŽX ° 0 between i and i+1

2) X |i

3) X |i+1

- **Example:**

  X (position) is a covering perspective in the block/spring example.

Definition

Relative change from the perspective X:

PÝ $^{X}_{i,i+1}$ **if** $|\hat{P}(x)| > |P(x)|$

P || $^{X}_{i,i+1}$ **if** $|\hat{P}(x)| = |P(x)|$ **} between i and i+1**

Pß $^{X}_{i,i+1}$ **if** $|\hat{P}(x)| < |P(x)|$

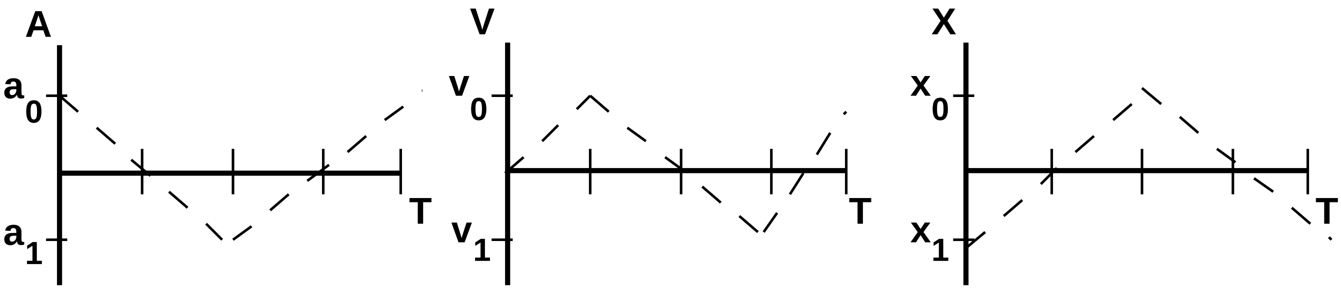
Solving the two block/spring problems

Assuming M is increased:

X does not change	*(self-reference rule)*
K does not change	*(interval constant rule)*
F equals -K times X	
So F does not change	*(multiplication rule)*

and

M increases	*(interval constant rule)*
F equals M times A	
So A decreases	*(multiplication rule)*
So V decreases	*(derivative rule)*
So the time duration increases	*(duration rule)*



Derivative rule:

A=dV/dt, V=dX/dt.
X covering perspective over (i,i+1).
A and V be positive over (i,i+1).

If not(V Ý$_i$), A ß$_{i+1}$ and A ß$^X_{(i,i+1)}$, then V Ý$^X_{(i,i+1)}$.

The Duration Rule

Definition of DISTANCE-BY:

X is increasing and positive (or decreasing and negative) over (i,i+1). DISTANCE-BY X over (i,i+1) is the relative change of the distance traveled over the interval:

		Starting RC value		
		Ý	Ý	Ý
Ending RC value	Ý	?	Ý	Ý
	||	ß	||	Ý
	ß	ß	ß	?

Duration rule:

X covering perspective over (i,i+1).
V=dX/dt
V ß $^{X}_{(i,i+1)}$
not (DISTANCE-BY X ß $_{(i,i+1)}$)

Then

$$\hat{T}(i+1) - T(i) \hat{>} T(i+1) - T(i),$$

i.e. the duration of (i,i+1) will increase.

Back to Quantities

A Qualitative Calculus Based on Signs

S = { + , 0 , - , ? }

Addition and multiplication:

+	0	+	-	?
0	0	+	-	?
+	+	+	?	?
-	-	?	-	?
?	?	?	?	?

*	0	+	-	?
0	0	0	0	0
+	0	+	-	?
-	0	-	+	?
?	0	?	?	?

"Qualitatively Equal"

$a \sim b$ iff $a = b$ or $a = ?$ or $b = ?$

Transformation rules:

$$[a + b] \sim [a] + [b]$$
$$[a * b] = [a] * [b]$$

Example:

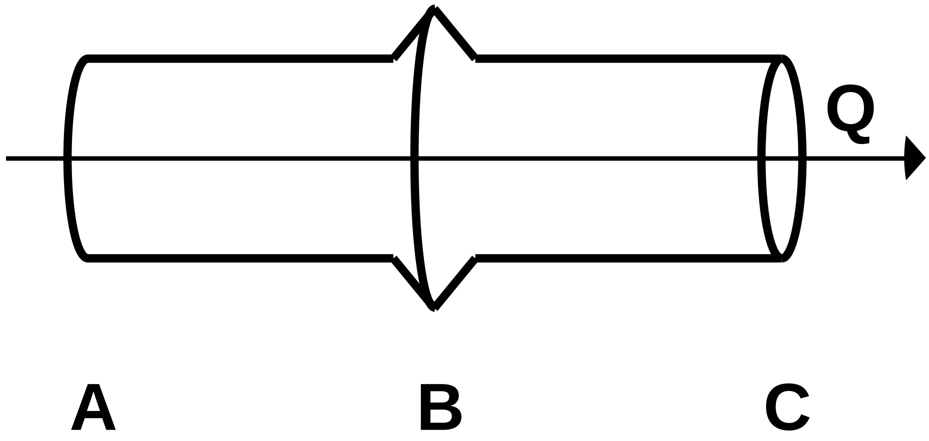
?P - ?Q ~ ?A

If ?P = + and ?Q = +
Then ?A = +, 0 or - are solutions.

Insight:

Physical quantities cannot take value ?

The Propagation Rules: Stuck



$[dP_A] - [dP_B] - [dQ] \sim 0$ (1)
$[dP_B] - [dP_C] - [dQ] \sim 0$ (2)

Case $[dP_A] = +$ and $[dP_C] = 0$

Propagation rule 1:
When a variable is assigned, substitute its value.

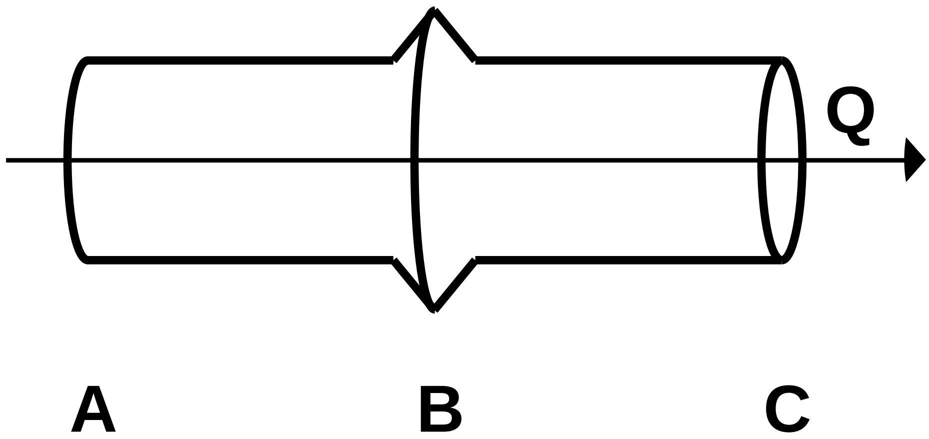
Propagation rule 2:
When a confluence involves a single variable, compute its value.

Applying propagation rule 1 leads to:

$- [dP_B] - [dQ] \sim -$ (1)
$[dP_B] - [dQ] \sim 0$ (2)

Stuck: Is ?Q +, 0 or - *(not ?)*

The Sum of Two Pipes is a Pipe



Two connected pipes

$$?P_A - ?P_C - ?Q \sim 0 \quad (3)$$

Case

$?P_A = +$, $?P_C = 0$

Then

$?Q = +$

Insight: Assembling a device

The Qualitative Resolution Rule

	x + y	˜	*a*	*(1)*
	-x + z	˜	*b*	*(2)*
If <u>x ??</u>, then				
	y + z	˜	*a + b*	*(3)*

x is a physical quantity. <u>Thus x ? ?</u>

Thus resolution applies to physical quantities

<u>Elimination:</u>

"One can eliminate a variable by adding or subtracting two confluences, provided that no other variable is eliminated at the same time."

<u>Negative example:</u>

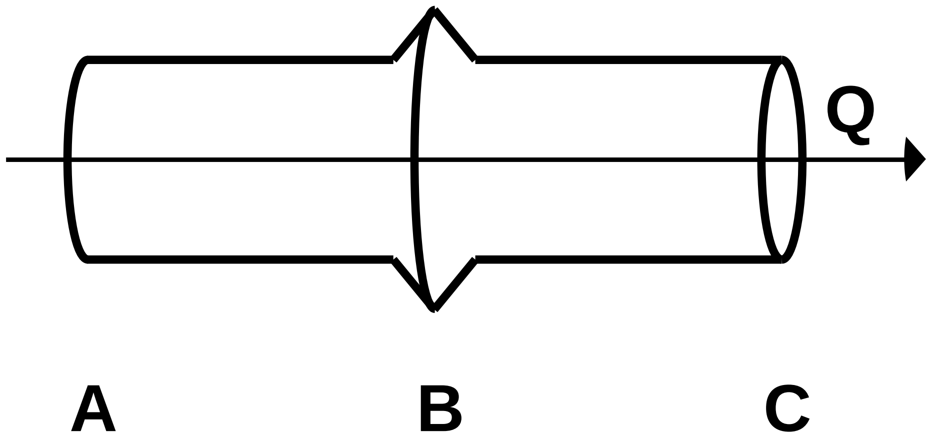
x + y + z + t˜ 0

x - y - z ˜ 0

does not implies

x + t ˜ 0

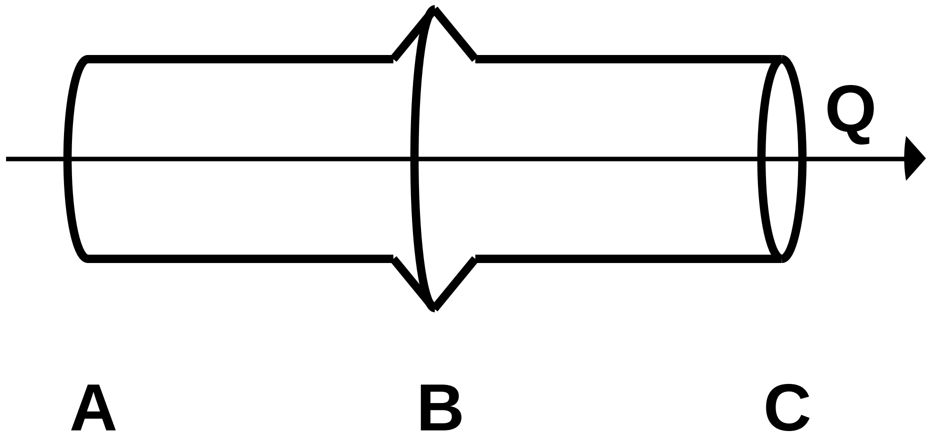
Example: Revisited



$?P_A$ - $?P_B$ - ?Q ˜ 0 (1)

\+ $?P_B$ - $?P_C$ - ?Q ˜ 0 (2)

⇒ $?P_A$ - $?P_C$ - ?Q ˜ 0 (3)



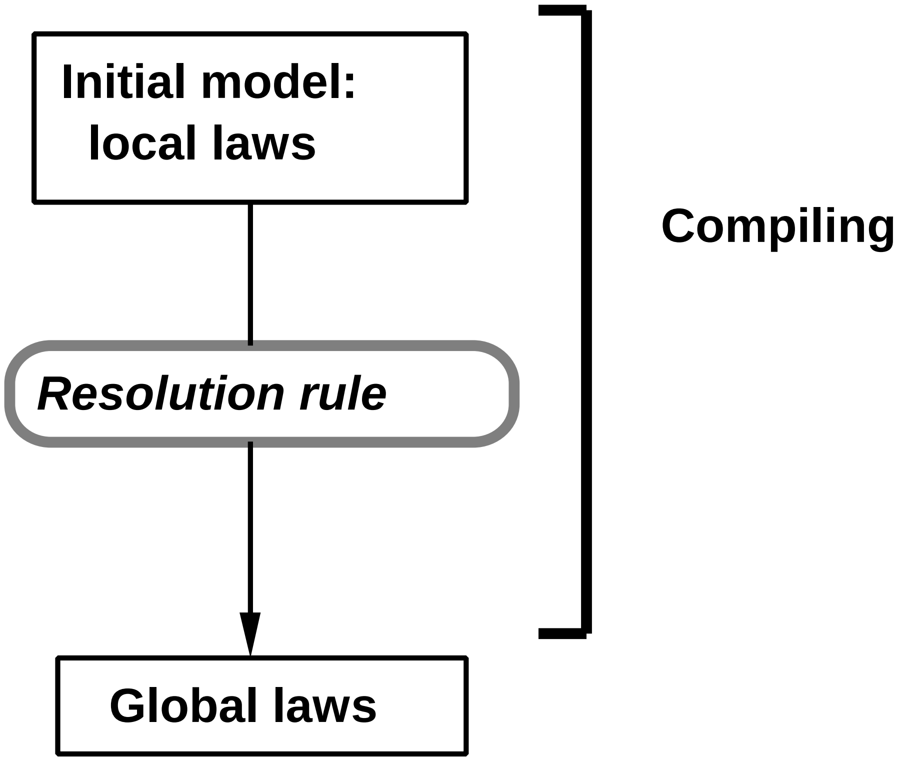
$?P_A - ?P_B - ?Q \; \tilde{} \; 0$ (1)

\- $?P_B - ?P_C - ?Q \; \tilde{} \; 0$ (2)

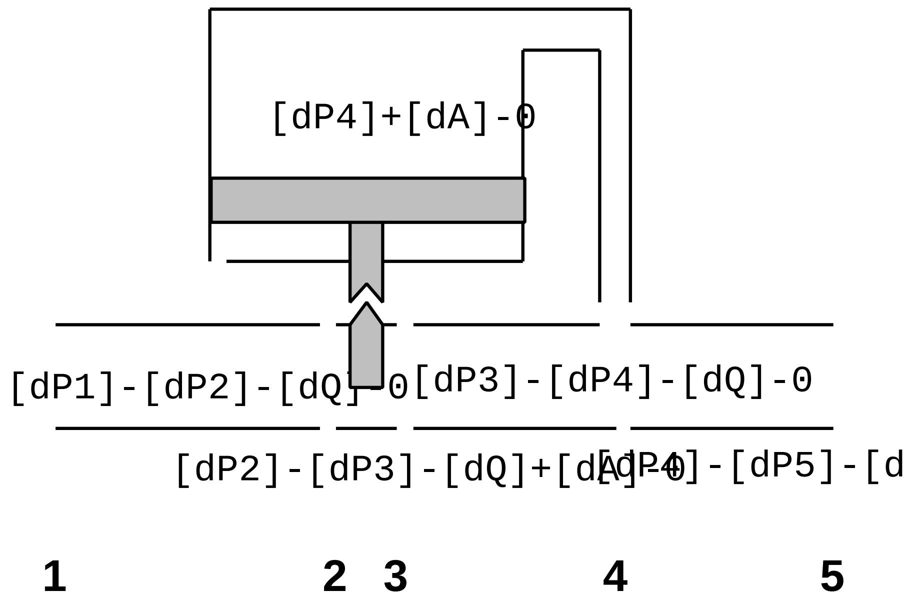
$\Rightarrow \quad ?P_A - ?P_B + ?P_C \tilde{} \; 0$ (4)

If $?P_A = +$ and $?P_C = 0$, then $?P_B = +$

Assembling a Device:
Two tasks



Second example: Pressure Regulator



$?P_1 -?P_2 -?Q$ ~ 0 (1)

$?P_2 -?P_3 -?Q +?A$ ~ 0 (2)

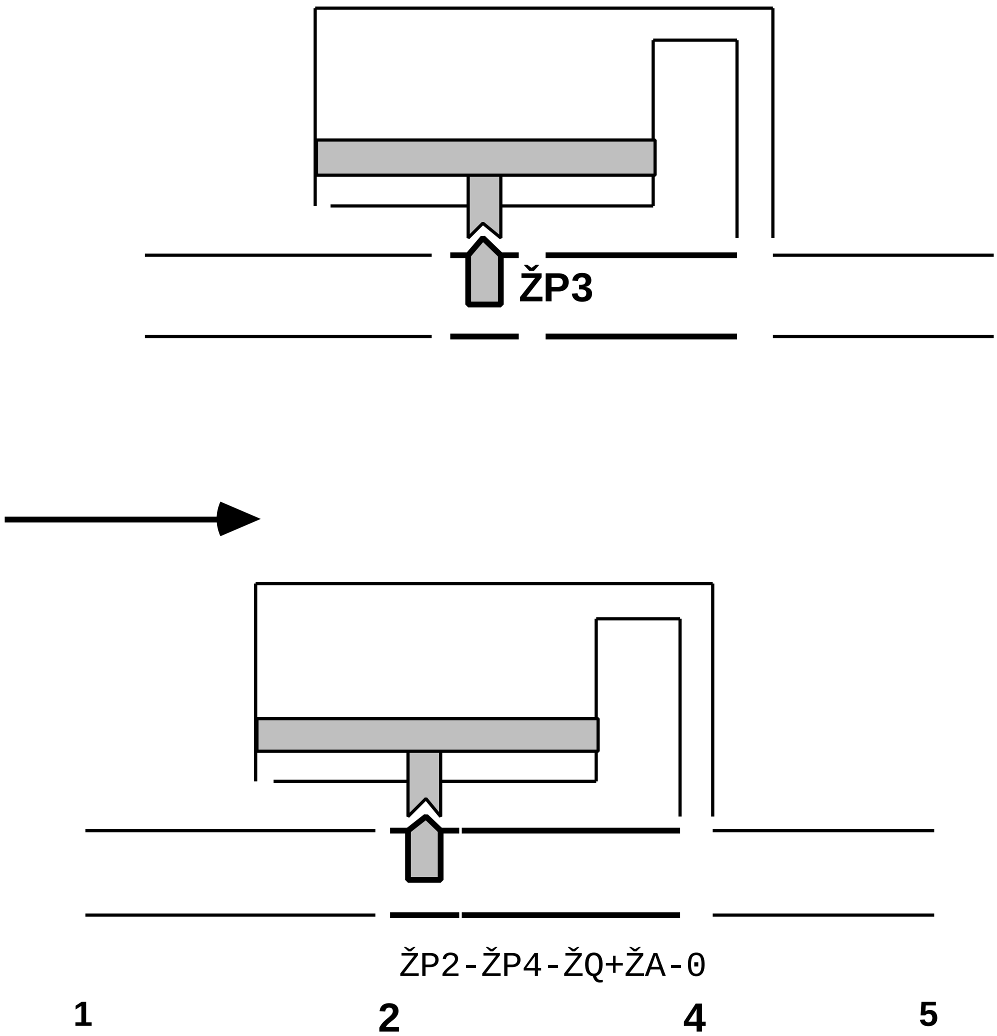
$?P_3 -?P_4 -?Q$ ~ 0 (3)

$?P_4 -?P_5 -?Q$ ~ 0 (4)

$?P_4 +?A$ ~ 0 (5)

Initial local model

From local to global



$?P_2$ -$?P_3$ -?Q +?A ˜ 0 (2)

$?P_3$ -$?P_4$ -?Q ˜ 0 (3)

$?P_2$ -$?P_4$ -?Q ˜ 0 (2)+(3)

Compiling

Elimination $\rightarrow$ Assemblages

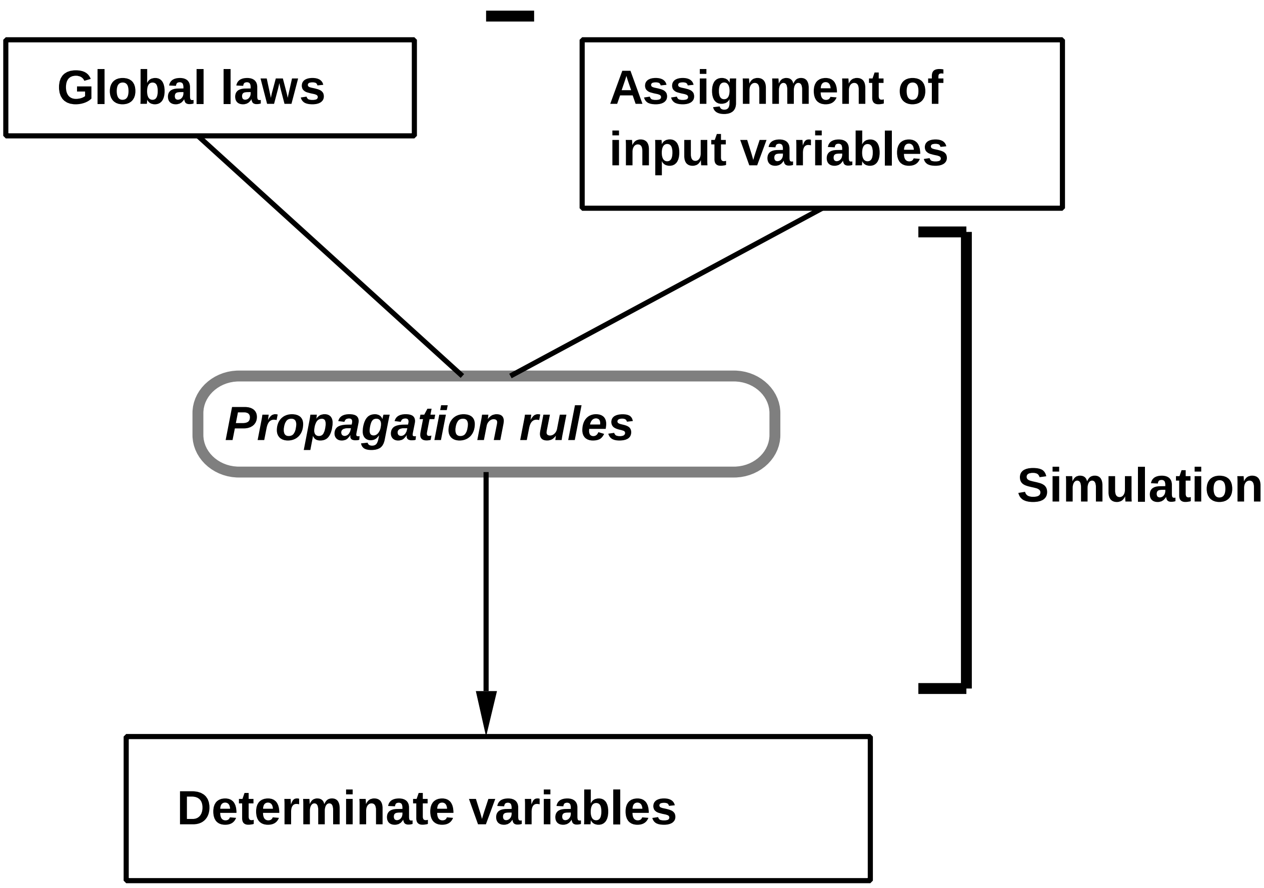
Assemblage for input variables

?P_2 ˜ ?P_1 + ?P_5 (SA_1)
?P_4 ˜ ?P_1 + ?P_5 (SA_2)
?A ˜ -?P_1 - ?P_5 (SA_3)
?Q ˜ ?P_1 - ?P_5 (SA_4)
?P_3 ˜ ?P_1 + ? ?P_5 (SA_5)

Assemblage for {?A,?Q}

?P_1 ˜ -?A + ?Q
?P_2 ˜ -?A + ?Q
?P_3 ˜ -?A + ?Q
?P_4˜ -?A
?P_5 ˜ -?A - ?Q

Simulation



$?P_1 \sim -?A + ?Q$

$?P_2 \sim -?A + ?Q$

$?P_3 \sim -?A + ?Q$

$?P_4 \sim -?A$

$?P_5 \sim -?A - ?Q$

Case

$?A = -, ?Q = +$

Then

$?P_1 = ?P_2 = ?P_3 = ?P_4 = +$

Completeness

$?P_1 \sim -?A + ?Q$

$?P_2 \sim -?A + ?Q$

$?P_3 \sim -?A + ?Q$

$?P_4 \sim -?A$

$?P_5 \sim -?A - ?Q$

Case

$?A = -, ?Q = +$

Then

$?P_1=?P_2=?P_3=?P_4=+$

$?P_5$ is ambiguous:

$?P_5 = +, 0$ and $-$ are solutions of the initial system

Insight:

All the determinate variables are obtained using simple propagation.

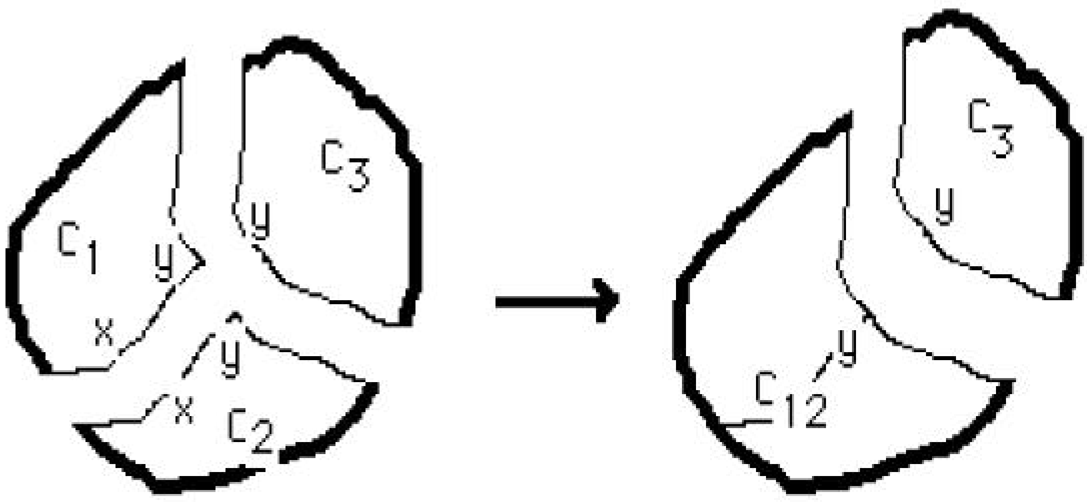
But ...

Freely applying the elimination rule leads to combinatorial explosion.

Pressure regulator (5 equations) --> hundreds of different ways for the resolution rule to apply

How to control qualitative resolution ?

Joining two components



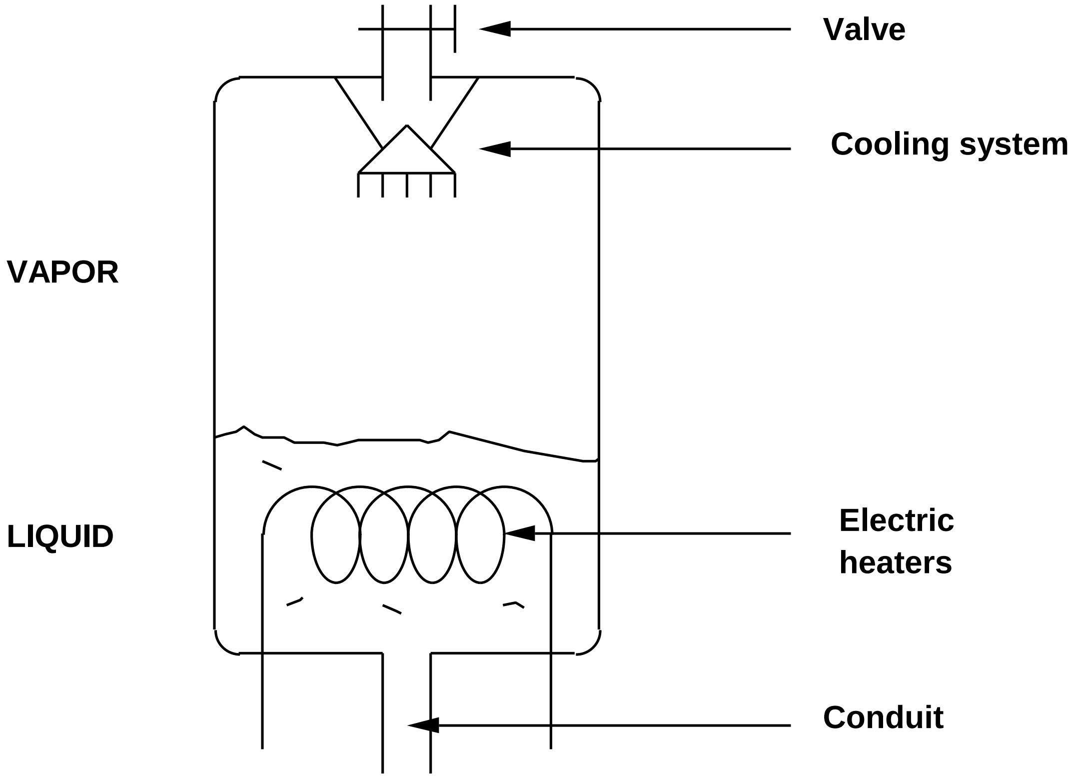
- y must appear in a model of C_{12}, but x should not.

- Joining rule: eliminate the variables (like x) involved only in two components.

- The joining rule is incomplete.

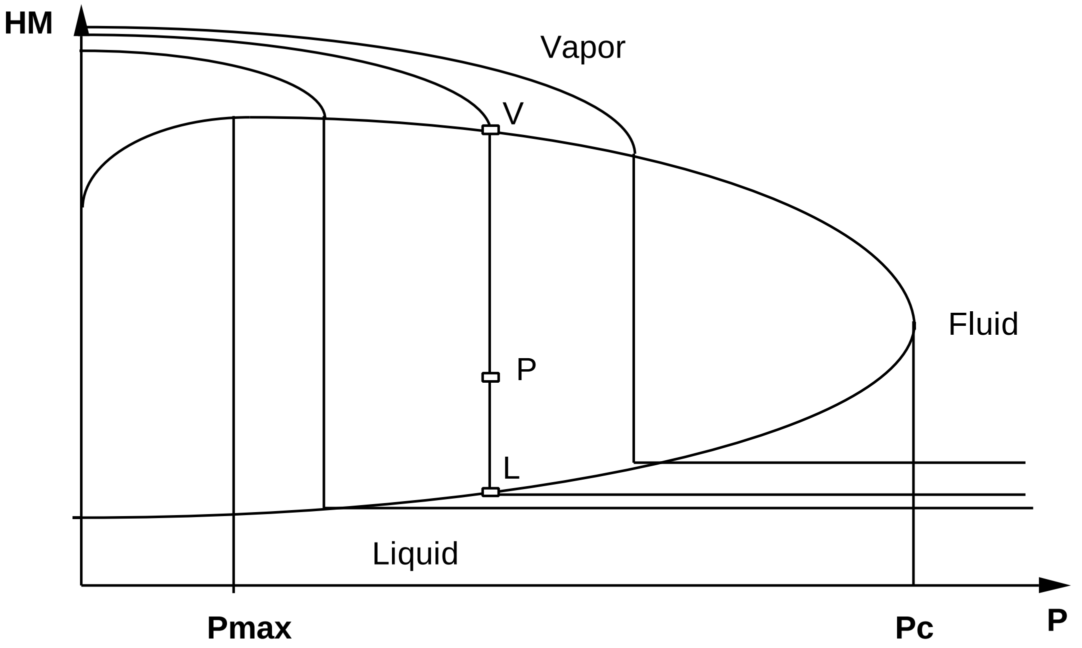
Building Qualitative Models When no Quantitative Equation is Available

The pressurizer of a nuclear power plant



The pressurizer of a nuclear power plant

Building a Qualitative Model:
A Qualitative Representation of Abacuses



The enthalpy-pressure diagram

Qualitative Model

Saturation curves

If Remains-Sat(Liq)
Then ?P - ?HM(Liq) ˜ 0 (7)

If Remains-Sat(Vap) and P<Pmax
Then ?HM(Vap) - ?P ˜ 0 (8)

If Remains-Sat(Vap) and P>Pmax
Then ?HM(Vap) + ?P ˜ 0 (9)

Regions under and over the saturation curves

If Is_Sat(Liq) and not Remains-Sat(Liq)
Then ?P - ?HM(Liq) ˜ + (10)

If Is_Sat(Vap) and not Remains-Sat(Vap) and P<Pmax
Then ?HM(Vap) - ?P ˜ + (11)

If Is_Sat(Vap) and not Remains-Sat(Vap) and P>Pmax
Then ?HM(Vap) + ?P ˜ + (12)

Isotherms: The temperature gradient

Qualitative equivalent of dT = –T(P,HM).(dP,dHM) (13)

We get:

?T(Liq) - ?HM(Liq) ˜ 0 (14)

?T(Vap) - ?P - ?HM(Vap) ˜ 0 (15)

Relative slopes of the isotherms and of the saturation curves for vapor

If Is_Sat(Vap) and Remains-Sat(Vap) and P >Pmax
Then ?T(Vap) - ?P + ?HM(Vap) ˜ 0 (16)

If Is_Sat(Vap) and not Remains-Sat(Vap) and P >Pmax
Then ?T(Vap) - ?P + ?HM(Vap) ˜ + (17)

Orders of Magnitude

Mechanisms

Appendix A:

Algebraic Properties of the Sign Algebra

Proof of the Resolution Rule

Proof

Quasi-transitivity of qualitative equality:

If

$$a \sim b \text{ and } b \sim c$$

and b ? ?

then

$$b \sim c$$

Compatibility of addition and qualitative equality:

$a + b \sim c$ is equivalent to $a \sim c - b$

Proof:

$x + y \sim a \rightarrow y - a \sim x$

$-x + z \sim b \rightarrow x \sim z - b$

$$\overline{y - a \sim z - b}$$

$$\rightarrow$$

$$y + z \sim a + b$$

Qualitative Linear Systems

• QLS = A qualitative linear system *not involving a quantity and one of its derivatives at the same time* (otherwise, one gets a *Qualitative Linear Differential System*).

• Solving a QLS

$AX \tilde{} B$

consists of finding vectors X without any ? component

• *Let X_0 be a solution of a QLS $AX \tilde{} B$. Then, for any real vector X'_0 with the sign pattern of X_0, there is a matrix A' and a vector B' with the sign patterns of A and B such that $A'X'_0 = B'$.*

• In practical terms, QLSs stem from:

? A set of real equations (possibly non-linear)

? A real differential system (comparative statics).

? A set of graphical constraints

Hard components

• For a real linear system:

? There is no solution

? There is a single solution

? There is an infinite number of solutions.

--> *The unicity problem is stated in terms of a global solution vector.*

• In a QLS, a component:

1) is a hard component

2) has solutions + and -, but not 0

3) has solutions +, 0 and -.

Qualitative Rank

• Independant qualitative vectors: *Let $V_1,...,V_n$ be some qualitative vectors of the same size. We say that they are independant iff for any $a_1,...,a_n$ all different from ?, the relation $a_1V_1+...+a_nV_n$ ˜ 0 implies $a_1=...=a_n=0$.*

• Qualitative rank:

? The rank of a qualitative matrix A is the maximum number of independant column vectors.

? A matrix A has full rank iff the QLS AX ˜ 0 has the single solution X=0.

? A QLS AX ˜ B is stationary iff matrix A has full rank.

• Qualitative rank and hard components: *Let AX ˜ B be a QLS with a hard component x_j. Then there is a subsystem with full rank involving x_j.*

Qualitative determinant

• Full rank and determinant: A square matrix A is not a full rank matrix iff Det(A)˜ 0.

• Qualitative Cramer's Formula: Let AX ˜ B be a non decomposable square QLS such that Det(A)?0. Let $A_{j/B}$ be the matrix deduced from A by substituting vector B for its j^{th} column.
Then, for any $\alpha_j \in \{+,0,-\}$ such that

$$\alpha_j \tilde{} \ \mathrm{Det}(A).\mathrm{Det}(A_{j/B}),$$

there exists a solution vector X such that its j^{th} component $x_j=\alpha_j$.

(A square matrix A is non decomposable if it cannot be matched by permuting its rows and columns to the form:

$$\begin{bmatrix} A_1 & 0 \\ B & A_2 \end{bmatrix}$$

when A_1 and A_2 are square matrices)

Signed
maximal
non decomposable
canonical
qualitative matrices

Signed = Determinant = + or - = Full rank

Maximal = The matrix becomes unsigned as soon as one replaces a 0 entry by a + or - entry.

Two matrices are equivalent iff they can be mapped on each other by composing the operators:

- exchanging two rows/two columns
- multiplying a row/a column by -

One selects a *canonical* representative from a class of equivalent matrices.

--> mathematical economists

```
+ - 0 . . . 0
+ + - 0 . . 0
.     . . .   .
.       . . . .
+       + + - 0
+ . . . + + -
+ . . . . + +
```

<u>Lancaster's matrices</u>

```
  N1    0
        N2
  0
+ + + + +
```

<u>Gorman's matrices</u>

```
0 + + +
+ 0 - +
+ + 0 -
+ - + 0
```

$$\begin{bmatrix} + & - & 0 & 0 & 0 \\ + & + & - & 0 & 0 \\ + & + & + & - & 0 \\ + & + & + & + & - \\ + & + & + & + & + \end{bmatrix} \quad \begin{bmatrix} + & - & 0 & 0 & 0 \\ + & + & - & 0 & 0 \\ + & + & + & - & - \\ 0 & 0 & 0 & + & - \\ + & + & + & + & + \end{bmatrix} \quad \begin{bmatrix} + & - & 0 & 0 & 0 \\ + & + & - & + & 0 \\ + & + & + & - & 0 \\ 0 & 0 & + & + & - \\ 0 & 0 & + & + & + \end{bmatrix}$$

$$\begin{bmatrix} + & - & 0 & 0 & 0 \\ + & + & - & - & 0 \\ + & + & + & - & 0 \\ + & + & 0 & + & - \\ + & + & 0 & + & + \end{bmatrix} \quad \begin{bmatrix} + & - & 0 & 0 & 0 \\ + & + & - & + & 0 \\ + & + & + & 0 & - \\ + & + & 0 & + & - \\ 0 & 0 & + & + & + \end{bmatrix} \quad \begin{bmatrix} + & - & - & 0 & 0 \\ + & + & 0 & + & 0 \\ + & 0 & + & 0 & - \\ - & 0 & 0 & + & - \\ 0 & - & + & + & + \end{bmatrix}$$

The six 5x5 signed maximal non decomposable qualitative matrices

Appendix B:

Some Other Qualitative Algebras

Non standard qualitative models: Orders of magnitude

- Let (I,+,=) be a totally ordered commutative group, and $(e_i)_{i\in I}$ be some distinct objects.

- $S^* = \{+e_i, -e_i, ?e_i\}_{i\in I} \cup \{0\}$

- $s_1e_i + s_2e_j =$ s_1e_i if $i>j$
  s_2e_j if $i<j$
  $(s_1+s_2)e_i$ if $i=j$

  $x + 0 = 0 + x = x$

- $s_1e_i \,.\, s_2e_j = (s_1.s_2)e_{i+j}$
  $x.0 = 0.x = 0$

- $s_1e_i \sim s_2e_j$ iff $s_1=?$ and $i>j$
  or
  $s_2=?$ and $i<j$
  or
  $s_1 \sim s_2$ and $i=j$

The Qualitative Resolution Rule for Orders of Magnitude

Let x, y, z, a, b be in S^* such that

x + y ˜ a (1)

-x + z ˜ b (2)

If <u>x has the pattern se</u>|<u> and if s is different from ?</u>, then

y + z ˜ a + b (3)

Interval algebras

• Consider (E,⊥). One defines ⊥ on P(E) by

A⊥B = {a⊥b; a∈A and b∈B}

• This enables us to define + and * on the set of the real intervals I. One defines ˜ on I by

I ˜ J iff I

Ιφ ονε χονσιδερσ α συβσετ J of I, one defines

I⊥ȷJ = Min{K∈J; K∉I⊥J}

provided that this exists.

• (S,+,*,˜) is an interval algebra with

+ =]0,+8[- =]-8,0[

? =]-8,+8[0 = [0,0]

• But, an interval algebra often has awful properties (the addition may be not associative).

The Qualitative Resolution Rule for Interval Algebras

- *Let (J,+J,*J,˜) be an interval algebra, and let x, y, z, a, b be elements of J such that*

  x + y ˜ a (1)

  -x + z ˜ b (2)

Suppose that J is stable under intersection (i.e. that I If <u>x is minimal with respect to inclusion</u> (that is, there exists no x' belonging to J such that xÉx' and x?x'), then

y + z ˜ a + b (3)

Other models

Dubois & Prade, 1988

• One considers three objects S, M and L, which are intended to represent the intervals]0,sm[,]sm,ml[and]ml,+8 [(but the landmarks sm and ml are unknown).

• F = {Set of intervals generated by unioning and multiplying by - the intervals S, L and M} U {0}.

• One can define + in different ways, for instance

S + S = +

or

S + S = S U M

We choose the second definition if we know that 2sm < ml.

• In either case, there is a resolution rule. The condition on x is that it belongs to the set {0,S,M,L,-S,-M,-L} (i.e., is minimal with respect to inclusion).

What is *qualitative?*

Here I discovered water - a very different element from the green crawling scum that stank in the garden but. You could pump it in pure blue gulps out of the ground, you could swing on the pump handle and it came out sparkling like liquid sky. And it broke and ran and shone on the tiled floor, or quivered in a jug, or weighted your clothes with cold. You could drink it, draw with it, froth it with soap, swim beetles across it, or fly it in bubbles in the air. You could put your head in it, and open your eyes, and see the sides of the bucket buckle, and hear your caught breath roar, and work your mouth like a fish, and smell the lime from the ground. Substance of magic - which you could tear or wear, confine or scatter, or send down holes, but never burn or break or destroy.

-From "Cider with Rosie", by Laurie Lee