NEW METHODS
IN QUALITATIVE CALCULUS

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Abstract

A qualitative calculus based on signs was first developed by economists to overcome one major difficulty: we often face a lack of quantitative data. A number of methods such as comparative statics were proposed to solve qualitative-model-based problems.
A new interest has recently arisen in qualitative methods as applied to process control, control theory, and artificial intelligence (the author is himself involved in Qualitative Physics - a subfield of AI). New approaches, such as order of magnitude reasoning, have extended the scope of "what is qualitative". Beyond some aspects specific to these fields - for instance representing how humans reason on the behavior of a system is a major concern in AI - this has led to new results and methods for dealing with qualitative models. These results are likely to be applicable within any framework, including economical modelling.
This paper discusses these new theoretical aspects of qualitative calculus. We hope that it will provide a better insight into these mathematical-like topics and contribute to the emergence of a general theory as to "what is qualitative".

1 Introduction

The word *qualitative* has been used byeconomists for more than forty years as a synonym for *reasoning about signs*. A new interest has recently arisen in qualitative techniques in other fields, such as Control Theory and Artificial Intelligence. The first step consisted of developing models, tools and techniques for reasoning in the qualitative space {+,0,-,?}. This has led to new results, that we shall present in the second section of this paper. Among them, a significant result for theoretical as well as practical purposes seems to be the existence of a *qualitative resolution rule* (so called because of its similarity - including a completeness result - with resolution in logic).
However, this work is not restricted to completing the task initiated byeconomists. New frameworks have been developed for capturing other intuitive ideas of what can be called qualitative, for example *order of magnitude* reasoning. We provide an outline of these models in the third section. All of them share the same feature: they always involve some kind of resolution rule. Beyond this coincidence, there must be a unifying algebraic structure. We are currently attempting to figure it out.

2 The standard qualitative algebra

2.1 Confluences

Consider a simple macroeconomical simulation model:

$$Con = B_{11} + B_{12}*(W_1+W_2) + B_{13}*Profit + B_{14}*Profit(-1)$$
$$W_1 = K*W_1(-1) + B_{21} + B_{22}*(Income+Tax-W_2) + B_{23}*(Income+Tax-W_2)(-1) + B_{24}*Time$$
$$Income = Con + Invest + Gov - Tax$$
$$Profit = Income - W_1 - W_2$$

where

Con = Domestic consumption

Gov	= Public expenditures
Income	= Gross domestic product
Invest	= Investments
Profit	= Profit
Tax	= Tax
W_1	= Private wages
W_2	= Public wages

and B_{ij} and K are positive coefficients. Time stands for the current time period, and X(-1), where X is a variable, for the value of variable X at the previous time period.
We do not intend to discuss the meaning nor the accuracy of this model. We only use it as a pretext for introducing the concepts of qualitative reasoning based on signs.
assuming you have the authority to decide at time t to increase or a decrease the "governmental variables" (Gov, Invest, Tax, W_2), you would like to know the effects of your decisions on the economy of your country. This can be performed in two steps:

- replacing Income by Con+Invest+Gov-Tax and Profit by Con+Invest+Gov-Tax-W_1-W_2.
- considering the difference ? Con and ? W1 caused by the decision {? Gov,? Invest,? Tax,? W2} at time t with respect to a reference decision. If we denote C_{11}=1-B_{13} and A=B_{12}-B_{13}, we get:

$$-A.?W_1 + C_{11}.?Con = A.?W_2 + B_{13}.?Gov - B_{13}.?Tax + B_{13}.?Invest$$
$$?W_1 - B_{22}?Con = -B_{22}.?W_2 + B_{22}.?Gov + B_{22}.?Invest$$

It is difficult to go further into the deductions without assessing the remaining coefficients. Instead, we shall try to get some perhaps poorer information but starting from a weaker kind of knowledge: signs of quantities. Under the assumption C_{11}>0, and if we denote a=sign(A) and ?X=sign(? X) for every variable X, we can write the following relations, called *confluences*, or *qualitative equations*:

$$-a?W_1 + ?Con \sim a?W_2 + ?Gov - ?Tax + ?Invest \quad (1)$$
$$?W_1 - ?Con \sim -?W_2 + ?Gov + ?Invest \quad (2)$$

The formal definitions of what is involved here are given below. However, we can explain what we intuitively mean. For example, under the assumption a=-, and if we suppose that we increase the taxes, but that we keep the other variables at their reference value, we get:

$$?W_1 + ?Con \sim - \quad (1)$$
$$?W_1 - ?Con \sim 0 \quad (2)$$

It can be checked (and this is proved later on) that, in this case, $?W_1$=- and ?Con=-. In other words, increasing the taxes tends to cause a decrease in private wages and domestic consumption.
This kind of method has been known and studied by economists for more than forty years (see for example [Lancaster, 1966] [Jeffries, Klee & Van den Driessch, 1977] [Ritschard, 1983]). However, researchers working in the fields of Artificial Intelligence or Control Theory have developed new techniques for dealing with this kind of reasoning. We shall present them now.

2.2 A qualitative model based on signs

We need to define our algebraic notations properly. In qualitative calculus based on signs, one considers the set S={+,0,-,?}. The element ? in the set S is necessary to deal with addition: e.g. (+)+(-) is defined as ?. Addition in non ambiguous cases and multiplication are defined in table 1. [x] denotes the sign of a real x.

+	0	+	-	?
0	0	+	-	?
+	+	+	?	?
-	-	?	-	?
?	?	?	?	?

*	0	+	-	?
0	0	0	0	0
+	0	+	-	?
-	0	-	+	?
?	0	?	?	?

Table 1: addition and multiplication of signs

While the relation = is the usual equality, we define ˜ on S as follows: for any a and b belonging to S, a˜ b iff a=b or a=? or b=?. ˜ is called *sign compability* or *qualitative equality*. Basic properties of these notions are studied in [Dormoy, 1987]. -s - where s is an element of S - stands for (-)*s.

2.3 Qualitative linear systems

A system involving qualitative quantities, but not at the same time one quantity and one of its derivatives, is called a *qualitative linear system* (QLS). If some quantity and one of its derivative are involved at the same time, then the system is called a *linear qualitative differential system* (QLDS). Qualitative vectors and matrices as well as addition and multiplication are clearly defined. All the entries of qualitative vectors or matrices appearing in a QLS or a QLDS are in DS={+,0,-}. Two vectors or matrices of the same size are sign compatible iff all their respective components are sign compatible. This relation will also be denoted ˜ .
Simpler issues must be tackled first. Qualitative linear differential systems represent a difficult topic currently under investigation by numerous researchers in Qualitative Physics (see also on sign stability [Jeffries, Klee & Van den Driessch, 1977]). The scope of this paper will be restricted to qualitative linear systems.
Practically, the components in a qualitative system are signs of real quantities. Therefore solving a QLS AX˜ B consists of finding vectors X without any ? component.

2.4 The link between qualitative and quantitative

If we consider a real linear relation A'X'=B', then the relation [A'][X']˜ [B'] is true as well. As far as quantitative linear systems are concerned, the converse is true in the following sense [Travé & Kaszkurewicz, 1986a, 1986b] [Dormoy, 1987]:

> *Let X_0 be a solution of a QLS AX˜ B. Then, for any real vector X'_0 with the sign pattern of X_0, there is a matrix A' and a vector B', with the sign patterns of A and B respectively, such that $A'X'_0=B'$.*

This property is theoretically important: it states that if we only know the signs of the entries of a quantitative linear system, then all the information we can get is contained in the corresponding QLS. This property is not true for some other qualitative models (for instance intervals algebra, see section 3 and [Struss, 1987]).
In practical terms, we have to deal with non-linear real systems more often than not. But even in this case, the qualitative behavior is described by a QLS. This is a great advantage of qualitative models: switching from quantitative to qualitative makes the system linear. Unfortunately, even if any real solution provides a qualitative solution, the converse is not true in general. This topic has not been studied yet.

2.5 Hard components.

For any real linear system, there are 3 mutually exclusive possibilities:

- there is no solution,
- there is a single solution,
- there is an infinite number of solutions.

In particular, the unicity problem is stated in terms of a global solution vector.

Now, consider example 2.1. Under the assumptions a=-, ?Gov=+, and the other decision variables being 0, it can be proved that $?W_1$=+. But ?Con remains ambiguous. Ambiguity is a well-known feature of qualitative models. But it does not necessarily concern the whole solution vector: some components may be well-defined while the others remain ambiguous.
Hence the qualitative case is radically different from the quantitative one. The notion of a hard component, namely a component of X which is perfectly determined by the set of confluences, turns out to be crucial.
In general, it can be proved [Dormoy, 1987] that, when at least one solution vector exists, there are exactly three possibilities for each component:

1) it is a hard component,
2) + and - are solutions, but 0 is not,
3) +, 0 and - are solutions.

Case 2 may look quite strange: a variable is ambiguous, but it cannot be 0. This often indicates a pathology: the system is not stationary, i.e. internal variables may be non-zero even when the input remains steady. An example of such a type of behavior is provided in example 2.1 when a=+: confluence (1) is changed to:

-$?W_1$ + ?Con ˜ $?W_2$ + ?Gov - ?Tax + ?Invest (1)

When $?W_2$ = ?Gov = ?Tax = ?Invest = 0, it can be proved that the solution set is $?W_1$ = ?Con = ± .

2.6 Qualitative rank

A "good" quantitative model is based on a set of independent equations. In particular, the model is stationary in the previous sense. We have shown that this property may be lost in the qualitative model.
A notion of qualitative independence was defined in [Travé & Kaszkurewicz, 1986a, 1986b]:

Let $V_1,...,V_n$ be some qualitative vectors of the same size. We say that they are independent iff for any $a_1,...,a_n$ all different from ?, the relation $a_1V_1+...+a_nV_n$˜ 0 implies $a_1=...=a_n=0$.

The rank of a qualitative matrix A is defined as the maximum number of its independent column-vectors. We say that A is a full rank matrix if its column-vectors are independent. A is a full rank matrix iff the QLS AX˜ 0 has the single global solution X=0. The concept of qualitative rank provides a tool for checking the stationarity property of a qualitative model.
Moreover, the following result connects the notions of rank and hard components [Travé & Kaszkurewicz, 1986a, 1986b] [Dormoy, 1987]:

Let AX˜ B be a QLS with a hard component x_j. Then there is a full rank subsystem involving x_j.

This proves that there is no hope of finding a hard component for a non stationary system with no stationary subsystem.

2.7 Qualitative determinant

It turns out that the previous notions and results are related in square systems to the qualitative determinant (the qualitative determinant of a square qualitative matrix A can be calculated as in the real case) (Dormoy, 1987):
Full rank and determinant: *Let A be a square qualitative matrix. A is not a full rank matrix iff Det(A)˜ 0.*
Qualitative Cramer's formula: *Let AX˜ B be a square QLS such that Det(A)?0, and x_j the j^{th} component of X.*

Let $A_{j/B}$ be the matrix deduced from A by substituting vector B for its j^{th} column, and Det($A_{j/B}$) its determinant. Let's assume that matrix A is not decomposable, i.e. cannot be matched by permuting its rows and columns to the form (A_1 and A_2 are square matrices):

$$\begin{bmatrix} A_1 & 0 \\ B & A_2 \end{bmatrix}$$

Then:

- *the QLS AX˜ B has at least one solution.*
- *the solution set for x_j is given by:*

Det(A) \ Det($A_{j/B}$)	+ or -	?
+ or -	{Det($A_{j/B}$).Det(A)}	{+,0,-}
?	{+,-}	{+,0,-}

2.8 The resolution rule

Finding out the hard components of a QLS is crucial for two reasons:

- it enables us to know the non ambiguous physical quantities.
- it reduces the search space a great deal.

The qualitative version of Cramer's formula is apparently a tool for this task. It is limited to square systems, but in practical terms the main reason for not using it is that it requires a huge amount of calculations.

However, the qualitative resolution rule [Dormoy, 1987] [Dormoy & Raiman, 1988] is an effective calculation tool:

Qualitative Resolution Rule: Let x, y, z, a, b be qualitative quantities such that

x + y ˜ a
and -x + z ˜ b
If x is different from ?, then
y + z ˜ a + b

Practically speaking, this rule means that a variable can be eliminated by adding or subtracting two equations provided that no other variable is eliminated at the same time.

Consider example 2.1 with a=-. Variable ?Con can be eliminated by adding confluences (1) and (2):

$?W_1$ + ?Con ˜ -$?W_2$ + ?Gov - ?Tax + ?Invest (1)
$?W_1$ - ?Con ˜ -$?W_2$ + ?Gov + ?Invest (2)

$?W_1$ ˜ -$?W_2$ + ?Gov - ?Tax + ?Invest (1) + (2)

Hence, $?W_1$ is a hard component as soon as -$?W_2$ + ?Gov - ?Tax + ?Invest ? ?. In the same way, by subtracting (2) from (1) we get:

?Con ˜ ??W_2 + ??Gov - ?Tax + ??Invest (1) - (2)

This means that ?Con is a hard component only when $?W_2$=?Gov=?Invest=0.

It can be proved in the square case [Dormoy, 1987] that the resolution rule is complete regarding the hard components problem: whenever a variable x is a hard component, the resolution rule finds this out and determines the value of x.

The resolution rule is a fundamental tool for solving QLS. It provides an equivalent of gaussian elimination in vector spaces for QLS. A set of heuristics can be defined for efficiently controlling in practical cases the solving process when using the resolution rule [Dormoy, 1987, 1988]. They prevent qualitative resolution from meeting the fate of resolution in logic.

This is not the only point. This rule is probably of dramatic theoretical importance. There are some versions of a resolution rule in other qualitative algebraic frameworks. We show them in section 3 (and we explain at the same time why we used the word "resolution"). The previously mentioned completeness result corroborates this impression.

2.9 Soft components

When the whole solution set of a QLS has to be determined, the resolution rule only solves part of the problem: it says nothing about the soft components, i.e. the ambiguous variables.

Several algorithms have been proposed to solve this problem [De Kleer & Brown, 1984] [Travé & Kaszkurewicz, 1986a, 1986b]. But as far as we know, the structure of the solutions of soft components has not been deeply investigated. We think that this would mean a great deal to the improvement of these algorithms.

3 Non standard qualitative models

We have shown in detail in the previous section some algebraic properties of qualitative models based on signs. We show here how one can model other intuitive notions.

3.1 Orders of magnitude

A model for order of magnitude reasoning is described in [Raiman, 1986]. It involves three relations between quantities: negligibility, closeness, comparability. We present here a weakened model, which extends the sign-based one. The comparability and negligibility relations are kept, but closeness is lost. Giving a complete picture of this model is necessary, albeit somewhat tedious.

Let (I,+,=) be a totally ordered commutative group (for instance the additive group of rational integers). Let e_i, $i \in I$, be some mutually distinct objects. We consider the set $S^*=\{+e_i,-e_i,?e_i\}_{i\in I\approx\{0\}}$. The e_i's are orders of magnitude, and we consider "signed orders of magnitude". Each element of S^* different from 0 can be written in a unique way as se_i, when $s \in S$ and $i \in I$. Moreover, $0e_i$ can be identified with 0 (this is consistent with the definition of multiplication given below). Addition, multiplication and the qualitative equality are defined on S^* in the following way:

Addition:

Let s_1, s_2 be two elements of S, both different from 0.

Let i, j, be two elements of I.

$s_1e_i+s_2e_j = s_1e_i$ if $i > j$

$= s_2e_j$ if $i < j$

$= (s_1+s_2)e_i$ if $i = j$, when s_1+s_2 represents the addition of s_1 and s_2 in S.

Let x be an element of S^*. Then $x + 0 = 0 + x = 0$

Multiplication:

Let s_1, s_2 be two elements of S and i, j two elements of I.

$s_1e_i.s_2e_j = (s_1.s_2)(e_{i+j})$, when $s_1.s_2$ represents the product of s_1 and s_2 in S.

Non standard qualitative equality:

Let s_1, s_2 be two elements of S, both different from 0. Let i, j be two elements of I.

$s_1e_i \sim s_2e_j$ iff $s_1 = ?$ and $i > j$

or

$s_2 = ?$ and $i < j$

or

$s_1 \sim s_2$ and $i = j$, when $s_1 \sim s_2$ means "s_1 and s_2 are qualitatively equal in the standard case".

Let x be an element of S^*.

$x \sim 0$ or $0 \sim x$ iff x=0 or x has the pattern $?e_i$ for one $i \in I$.

A short explanation: when subtracting two quantities having the same order of magnitude, it is possible to get a quantity having a strictly lower order of magnitude (just like when subtracting two standard quantities one can get 0). This is why $?e_2$ is qualitatively equal to e_1: $?e_2$ may derive from the subtraction of two quantities having the order of magnitude e_2, and then can be of any lower order of magnitude.

Let 0 denote the neutral element of the additive group (I,+). Then se_0 can be identified with s for any s in S. This is consistent with our definitions.

S^* can be viewed as an infinite stack of copies of S. Each level corresponds to an order of magnitude. S is embedded in this model: it is the basic level, corresponding to the e_0 order of magnitude. Hence S^* and its structure is a generalization of S.

3.2 Algebraic properties of orders of magnitude

As S is embedded in S*, any standard confluence is a non standard one as well. Most of the notions defined in previous sections within the standard framework also apply to the non standard one. The most interesting fact is that the resolution rule is sound within the non standard framework:

Non standard qualitative resolution rule: *Let x, y, z, a, b be any signed orders of magnitude such that the following relations hold:*

$x + y \sim a$ (1)

$-x + z \sim b$ (2)

If x has the pattern se_i *and s is different from ?, then:*

$y + z \sim a + b$ (3) = (1)+(2)

No completeness result concerning non standard resolution has been proved so far. But we expect that there is one.

3.3 Interval algebras

Interval algebras is another kind of qualitative model. It has been invertigated earlier than the previous ones. Indeed, the sign-based model is a special kind of interval algebra.

The underlying motive for studying interval algebras is that we might not know the exact value of a coefficient we are interested in, but an interval wherein it lies. Moreover, we may only be interested in comparing quantities with some special landmarks. This justifies the following definitions.

Consider a set E handled with a composition law ⊥. We can define a composition law (also denoted ⊥) on P(E) by A⊥B={a⊥b; a∈A and b∈B}. This enables us to define addition and multiplication on the set I of real intervals: if we consider two intervals I and J of R, then I+J and I*J are also intervals. The compatibility relation ˜ is defined on I by I˜J iff IΤηε τρουβλε ωιτη ιντερϖαλ αλγεβρασ ισ τηατ τηεψ ϖερψ οφτεν ηαϖε δρεαδφυλ προπερτιεσ. Φορ εξ αμπλε, τηερε ισ νο ρεασον φορ αδδιτιον το βε ασσοχιατιϖε! Ηοωεϖερ, σομε αττεμπτσ ηαϖε βεεν μαδ ε το βυιλδ σψστεμσ υσινγ τηεμ, φορ εξαμπλε τηε Κυιπερσ϶ ΘΣΨΜ σψστεμ [Κυιπερσ, 1984, 1986].

(S,+,*,˜) can be viewed as an interval algebra as soon as we define +=]0,+ [, 0=[0,0], -=]- ,0[and ?=]- ,+ [. Properties of interval algebras can be found in [Struss, 1987].

A resolution rule can be stated within the interval algebras framework:

Let $(J,+_J,*_J,\sim)$ *be an interval algebra, and let x, y, z, a, b be elements of J such that:*

$x + y \sim a$

$-x + z \sim b$

Suppose that J is stable under intersection (i.e., that I *y + z Å a + b*

3.4 Other models

Dubois and Prade proposed in [1988] a new model for some kind of order of magnitude reasoning. The main difference with the previous orders of magnitude model is that, when adding two small quantities, one may get a quantity which is not small.

Let's consider three objets S (for small), M (for medium) and L (for large). They are intended to represent three intervals]0,sm[,]sm,ml[and]ml,+ [, but bounds sm and ml are unknown. We consider the set F made up of S, M, L, the composite objects SM, ML and + corresponding to the union of two or three of these intervals, their negative equivalent and again the resulting intervals stemming from a combination of negative and positive intervals. Addition and multiplication are defined on F by:

$I \perp_F J$ is the minimal K belonging to F such that, for any sm and ml, K€I⊥J (where I⊥J is taken in I).

For instance, S+M=M+L=+, S+L=M+L=L+L=++L=+, ... This definition is different from the one in interval arithmetic. It is essentially based on the fact that sm and ml are unknown. F is not isomorphic to any interval algebra.

As in interval algebras, ˜ is defined by I˜J iff IΤηερε ισ α νεω ϖερσιον οφ τηε ρεσολυτιον ρυλε ωιτηιν τηισ φραμεωορκ. Τηε χονδιτιον ον ξ ισ τηατ ιτ βελονγσ το τηε σετ {Σ,Λ,Μ,0,–Σ,–Μ,–Λ}, τηατ ισ το τηε σετ οφ ελεμεντσ οφ Φ μινιμαλ ωιτη ρεσπεχτ το ινχλυσιον.

3.5 What is *qualitative*?

We have not explained yet why we are using the word *resolution*. The qualitative resolution rule and the resolution rule in logic (weakened here to the propositional calculus) have a similar aspect:

Let X, Y, Z be propositional variables (and x, y, z their boolean equivalents) such that

X v Y (*x + y = 1*)

and X v Z (*x + z = 1*)

Then

Y v Z (*y + z = 1*)

Moreover, the resolution rule in logic as well as the one within the sign-based framework have completeness properties. We are thus facing a situation with similar algebraic structures and similar rules (plus two completeness results) in models of increasing complexity: there is something fishy going on. We have not grasped it yet. What we may catch is a general algebraic structure capturing the idea of what is *qualitative*.

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